Lesson 13h: Kirchoff's Rules

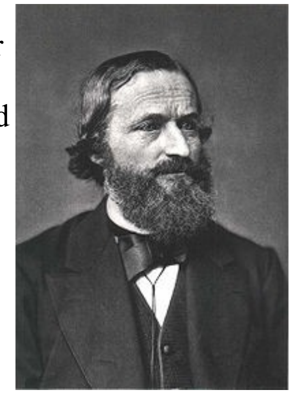


Illustration 1: G.R. Kirchhoff

We sometimes encounter a circuit that is too complicated for simple analysis.

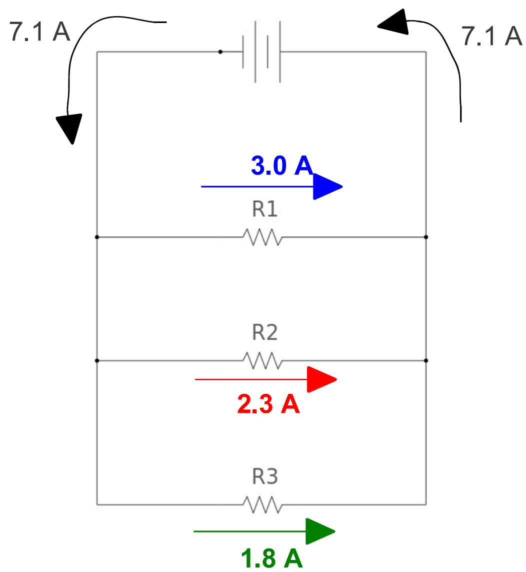
- Maybe there is a weird mix of series and parallel, or more than one power source.
- To deal with such complicated circuits, we use Kirchhoff's rules, invented by G.R. Kirchhoff (1824-1887).
- His rules are just convenient applications of the laws of conservation of charge and energy.
- In the previous section, you were already (sort of) using his rules, even if you didn't know it.

The First Rule: The Junction Rule

Kirchhoff's first rule is based on the conservation of charge, and we already used it in deriving the rule for parallel resistors.

"At any junction point, the sum of all currents entering the junction must equal the sum of all currents leaving the junction."

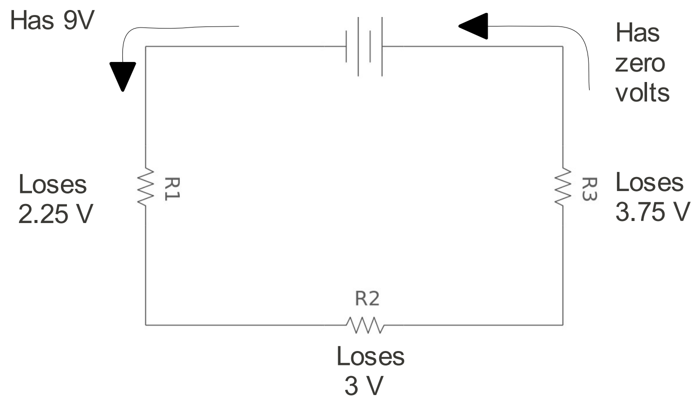
- This just means that when current reaches the branches in a parallel circuit, it will split up and take different routes.
- When the branches come back together, the currents will add back together too.



The Second Rule: The Loop Rule

This one is based on the conservation of energy.

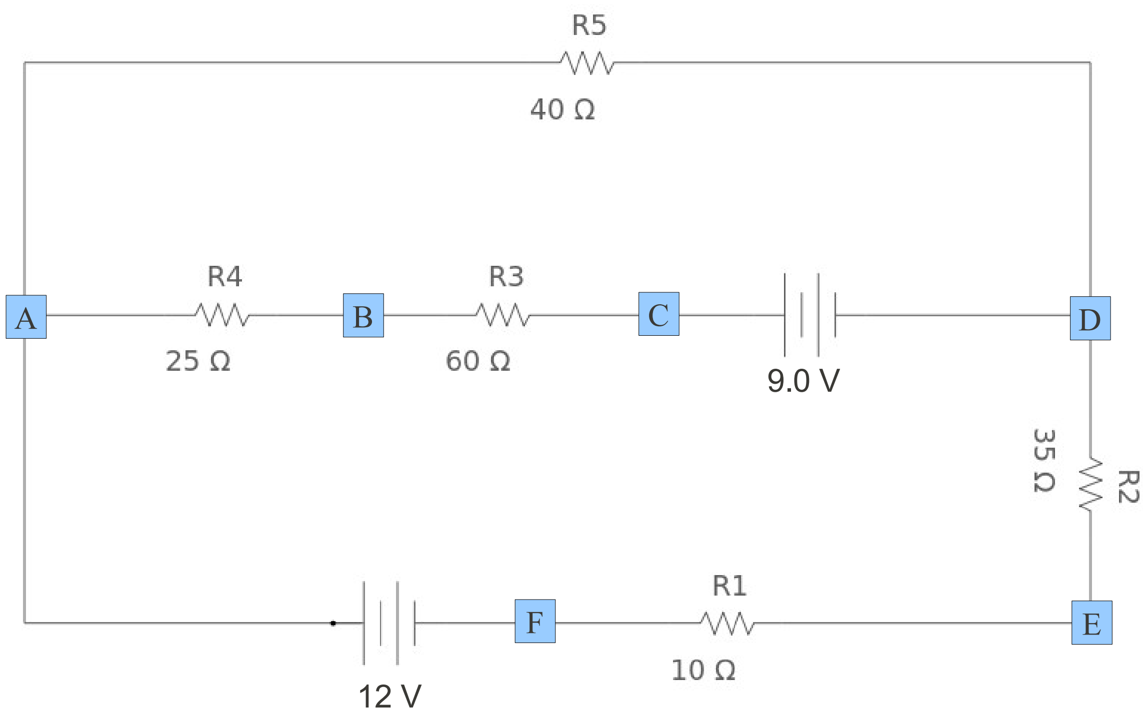
"The algebraic sum of the changes in potential around any closed path of a circuit must be zero."



This just means that when you look at the resistors in series, the drop in voltage across all of them will be equal to whatever the source is.

The importance of these rules comes when you need to analyze a more complicated circuit that has a combination of series and parallel circuits parts, along with more than one voltage source.

Example 1: **Determine** the current and voltage drops in all parts of this circuit. (The letters in boxes are drawn simply for reference and have nothing to do with the circuit. They will be explained in the solution.)

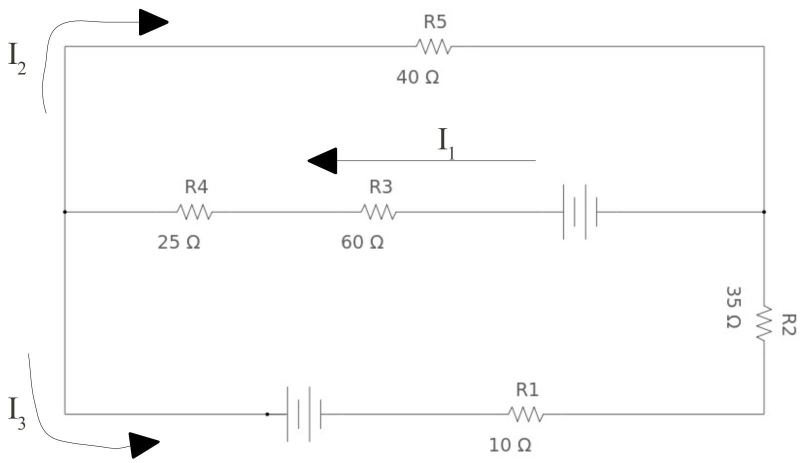


To figure this out, you will need to calculate the voltage drops in each part of the circuit, while tracing the current flowing through each part.

- I've put in some letters to divide up the parts of the circuit where something might happen to the voltage.
- Notice that in between the letters there is only one resistor or one battery... I've isolated the circuit into manageable parts.
- Since we are dealing with resistors (where voltage will drop) and batteries (where voltage will increase), we will need to use + and – to show when voltage has increased or decreased.
- We're going to need to trace two paths through this circuit, which will overlap on one of the branches.

Assume the currents are going in the direction shown here.

- Before you ask "How do we know this?" I'll just tell you we don't know!
- It's just a guess... all we need is to show currents going somehow through the circuit so we can start using the junction rule.



According to the junction rule and the diagram above, we are assuming...

$$I_1 = I_2 + I_3$$

- In the end if we find a value for one of these currents that is negative, it just means we drew the arrow pointing in the wrong direction.
- Now I can start writing down formulas for each section, using formulas like V =IR.
- Even though we may have some unknowns, we can use the labels drawn above to represent different currents in different branches.
- **Remember that the voltage change in any loop must equal zero!**

Part 1: ADCBA

This follows the path around I_2 and then through I_1, starting at position A.

$_aV_d = -I_2\,(40\Omega)$ ← put in the minus sign since I_2 **LOSES** potential across R_5

$_dV_c = -9V$ ← going the wrong way across the battery for an electron, so it will lose 9 V

$_cV_b = -I_1\,(60\Omega)$

$_bV_a = -I_1\,(25\Omega)$

Now we're back to where we started... point A on the diagram. Remember, the voltage drops across any complete path along a circuit must equal zero!

$0 = {}_aV_d + {}_dV_c + {}_cV_b + {}_bV_a$
$0 = -I_2\,(40\Omega) + 0 + -9 + -I_1\,(60\Omega) + -I_1\,(25\Omega)$ ← Simplify all this to get the next line
$0 = -40\,I_2 + -85\,I_1 + -9$

Hint: I know where this problem is going, so just trust me for now and solve the equation above for I_2 .

- That way we can do some substitution later on into $I_1 = I_2 + I_3$
- You don't always have to do them this way.

$$0 = -40\,I_2 + -85\,I_1 + -9$$
$$40\,I_2 = -85\,I_1 + -9$$
$$I_2 = \frac{(-85\,I_1 + -9)}{40}$$
$$I_2 = -2.125\,I_1 + -0.225$$

Part 2: AFEDCBA

Do this one like Part 1. This follows the path around I_3 and then through I_1, starting at position A.

${}_aV_f = +12$ V ← current is going the correct way thru this battery, so it's an increase
${}_fV_e = -I_3\,(10\Omega)$
${}_eV_d = -I_3\,(35\Omega)$
${}_dV_c = -\,9$ V
${}_cV_b = -I_1\,(60\Omega)$
${}_bV_a = -I_1\,(25\Omega)$

$0 = {}_aV_f + {}_fV_e + {}_eV_d + {}_dV_c + {}_cV_b + {}_bV_a$
$0 = +12 + -I_3\,(10\Omega) + -I_3\,(35\Omega) + -9 + -I_1\,(60\Omega) + -I_1\,(25\Omega)$ ← Simplify...
$0 = -85\,I_1 + -45\,I_3 + 3$

Solve for I_3. Again, trust me.

$$0 = -85\,I_1 + -45\,I_3 + 3$$
$$45\,I_3 = -85\,I_1 + 3$$
$$I_3 = \frac{(-85\,I_1 + 3)}{45}$$
$$I_3 = -1.8889\,I_1 + 0.06667$$

Part 3: Solve $I_1 = I_2 + I_3$

Here's why it was a good idea to solve for I_2 and I_3; now we can just substitute them in to the formula we have for current and solve for I_1.

$$I_1 = I_2 + I_3$$
$$I_1 = (-2.125\, I_1 + -0.225) + (-1.8889\, I_1 + 0.06667)$$
$$I_1 = -4.014\, I_1 + -0.1583$$
$$5.014\, I_1 = -0.1583$$
$$I_1 = -0.032\, A$$

The minus sign means we drew our arrow for I_1 in the wrong direction.

Substitute the value you just got for I_1 into the relationship for I_2...

$I_2 = -2.125\ I_1 + -0.225$
$I_2 = -2.125\,(-0.032) + -0.225$
$I_2 = -0.16A$

The minus sign means we drew our arrow for I2 in the wrong direction.

Substitute the value you got for I_1 into the relationship for I_3...

$I_3 = -1.8\ I_1 + 0.6$
$I_3 = -1.8\ (-0.032) + 0.6$
$I_3 = 0.13A$

Groovy! We actually drew that arrow in the correct direction.

Since you now know the currents in each part, you could also calculate the voltage drops in each part using V = IR. Go ahead and try... the answers are:

${}_aV_d = 6.4V$
${}_cV_b = 1.92V$
${}_bV_a = 0.8V$
${}_fV_e = 1.3V$
${}_eV_d = 4.55V$