Lesson 35: Compton Effect

The photoelectric effect and Einstein's theories about light having a particle nature caused a lot of scientists to start to reexamine some basic ideas, as well as come up with some new ones.

- Based on a lot of Einstein's work (including his Special Theory of Relativity), physicists predicted that these photons should have momentum, just like other particles do.
  - The tough part is explaining and proving this in a reasonable way, since you don't exactly feel light hammering you into the floor.
  - The momentum that the light photons have must be very small, and not based on the common way of calculating momentum using p = mv (since light has no rest mass).
  - Instead the formula was based on the wavelength and frequency of the light, just like Planck's formula.

Do you find it kind of odd that we're talking about all this stuff supporting the theory that light acts like a particle, and yet still doing calculations using values for wavelength and frequency? Sounds kinda wave-like to me. I don't blame you. This is wave-particle duality. At times we can't really separate the two from each other.

$$p = \frac{h}{\lambda} \quad \text{or} \quad p = \frac{hf}{c}$$

p = momentum (kgm/s)
h = Planck's Constant (always 6.63e-34)
λ = wavelength (m)
f = frequency (Hz)
c = speed of light

Warning!
Only the first formula is on your data sheet, but the other one is just derived from substituting c = f λ. Be careful to only use these formulas if you are doing something involving the momentum of photons.

In 1923 A.H. Compton started shooting high frequency x-rays at various materials and found that his results seemed to support the idea of photons having momentum. In one setup he shot the high frequency x-rays at a piece of graphite.

Graphite is just a hunk of solid carbon.

- If light was a wave, we would expect the x-rays to come out the other side with their wavelength smaller (*Illustration 1*).
  - Basically we can explain this as the waves squishing when they hit the graphite, like a ball squishing when it hits the ground.

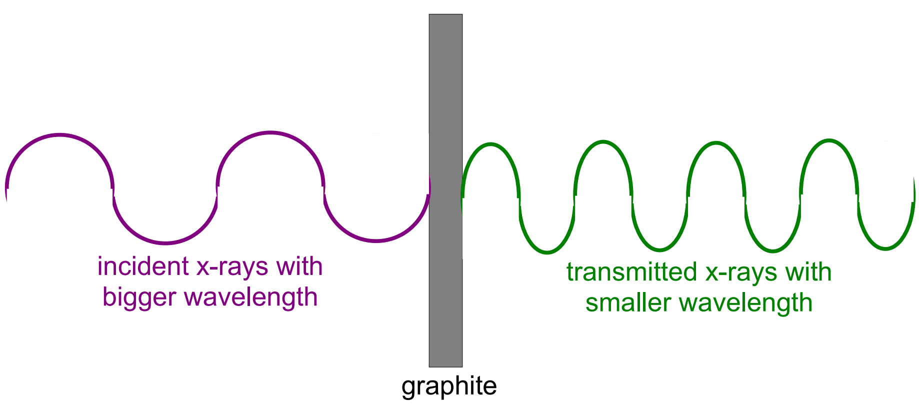


Illustration 1: If light was a wave, we'd expect results like this.

- Instead, Compton found that the x-rays scattered after hitting the target, changing the *direction* they were moving and actually getting a *longer* wavelength.
  - Remember, longer wavelength means smaller frequency.
  - Since E = hf, the scattered photons had less energy! Somehow, the x-ray photons were losing energy going through the graphite. So where'd the energy go?

Compton found that electrons were being thrown off the target at an angle.

- Compton was able to explain all he was seeing (which became known as the **Compton Effect**) by using
  - The conservation of energy (the energy the photon lost had to go somewhere).
  - The photon theory of light (to figure out the momentum of the photons).
  - The conservation of momentum (to explain the angles things were shooting off at).

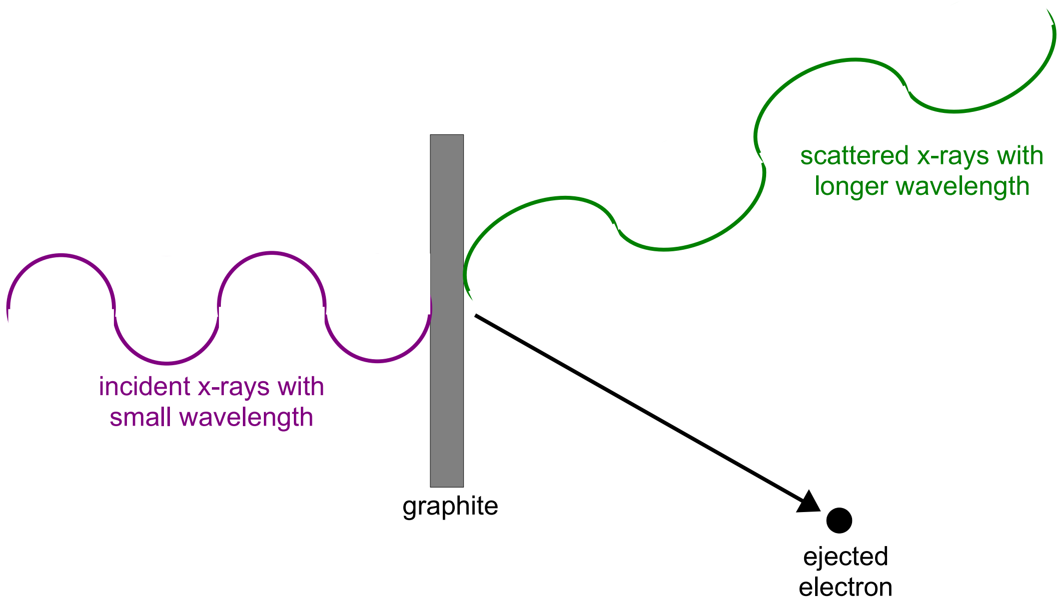


Illustration 2: What Compton actually observed.

If we looked at this in terms of momentum, we'd need to be careful about using the correct momentum equations for each part.

- **Incident x-rays**

  The original x-rays have a small wavelength, and the formula $p=\frac{h}{\lambda}$ shows us that this means it has a lot of momentum.

- **Scattered x-rays**

  The x-rays that made it through have a bigger wavelength, so $p=\frac{h}{\lambda}$ means it has less momentum.

- **Ejected electrons**

  We would calculate the electron's momentum using a classic p=mv calculation.

Compton found that if he did all the calculations, using the ideas of treating **light like photons** (to figure out their momentum) and **conservation of momentum** in a 2D collision, the numbers worked beautifully.

- In fact, it's almost spooky just how perfectly the numbers worked out. It was basically a 100% perfect conservation of momentum.
- This showed that the particle model of light with all its talk about photons must be correct.
  - This was a turning point in the particle theory of light, when the majority of physicists started to really believe the wave-particle duality of light was probably correct.

But you might also be saying to yourself "Hey, we've seen this all before… it's just the photoelectric effect!" That would be wrong.

Photoelectric Effect	**Compton Effect**
High frequency EMR hits metal.	High frequency EMR hits **non-metal**.
Conservation of energy occurs.	Conservation of **momentum** occurs.

After looking at the data he got from his experiments, Compton also found that he could predict the exact change in the wavelength between the incident and scattered x-rays.

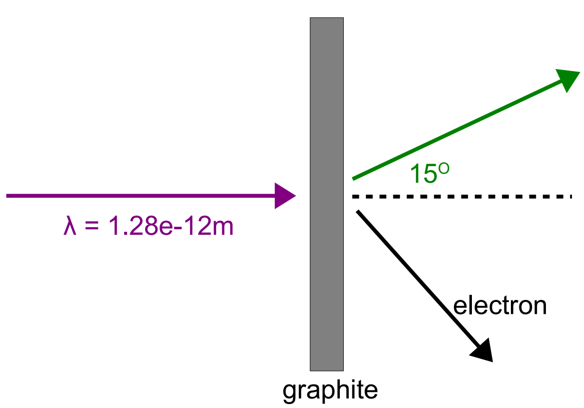
- His formula was based on the wave-particle duality of light, as well as the angle of the scattered x-ray.

$$\Delta\lambda = \frac{h}{mc}(1-\cos\theta)$$

$\Delta\lambda$ = change in wavelength between incident and scattered (m)
h = Planck's Constant
m = mass of electron (kg)
c = speed of light
θ = scattered x-ray's angle

Example 1: An x-ray light source with a wavelength of 1.28e-12m is shot at a piece of graphite. On the other side, the x-ray is observed to have scattered at an angle of 15° away from the original path. **Determine** the wavelength of the scattered x-ray (ignore sig digs). Then **determine** the velocity of the ejected electron.

First, draw a sketch just to make sure you have everything correct. Rather than go to the trouble of drawing the x-rays as waves, we'll just draw them is vectors to show their paths.



Then we'll figure out the scattered x-ray's wavelength...

$$\Delta\lambda = \frac{h}{mc}(1-\cos\theta)$$
$$\Delta\lambda = \frac{6.63\text{e-}34}{9.11\text{e-}31(3.00\text{e}8)}(1-\cos 15^o)$$
$$\Delta\lambda = 8.266072876\text{e-}14\,m$$

$$\Delta\lambda = \lambda_{final} + \lambda_{initial}$$
$$\Delta\lambda = \lambda_{scattered} + \lambda_{incident}$$
$$\lambda_{scattered} = \Delta\lambda + \lambda_{incident}$$
$$\lambda_{scattered} = 8.26607287\text{e-}14 + 1.28\text{e-}12\,m$$
$$\lambda_{scattered} = 1.362660729\text{e-}12\,m$$

We will handle the second part of the question like any other 2D conservation of momentum question, where we know stuff about two things and figure out the third.
Let's figure out the momentum (without rounding for sig digs yet) of the two x-rays...

Incident X-Ray

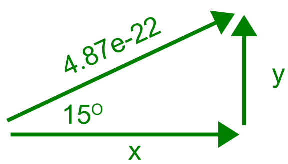
$$p = \frac{h}{\lambda}$$
$$p = \frac{6.63\text{e-}34}{1.28\text{e-}12}$$
$$p = 5.1796875\text{e-}22\,kgm/s$$

Scattered X-Ray

$$p = \frac{h}{\lambda}$$
$$p = \frac{6.63\text{e-}34}{1.362660729\text{e-}12}$$
$$p = 4.86548108422\text{e-}22\,kgm/s$$

The **incident x-ray** is all of the "before" momentum, and it's all along the x-axis. So the momentum before the collision is 5.18e-22 kgm/s.

We need to break up the **scattered x-ray** into x-axis and y-axis components, and use them to figure out the electron's momentum.



x-component

$$\cos\theta = \frac{adj}{hyp}$$
$$adj = \cos\theta(hyp)$$
$$adj = \cos 15^o(4.87\text{e-}22)$$
$$adj = 4.69969383657\text{e-}22$$

y-component

$$\sin\theta = \frac{opp}{hyp}$$
$$opp = \sin\theta(hyp)$$
$$opp = \sin 15^o(4.87\text{e-}22)$$
$$opp = 1.25927916818\text{e-}22$$

The **electron** has the rest of the x-axis momentum, and a y-axis momentum that cancels out the **scattered x-ray**...

x-component
5.1796875e-22 – 4.69969383657-22 = 4.79993663428e-23

y-component
-1.25927916818e-22

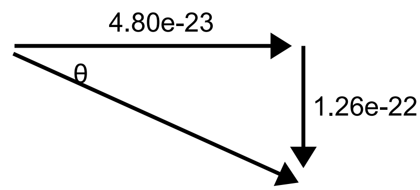


Figure out the angle and the hypotenuse, and then use it to get the velocity.

$$\tan\theta = \frac{opp}{adj}$$
$$\tan\theta = \frac{1.26\text{e-}22}{4.80\text{e-}23}$$
$$\theta = 69.13488379 = 69.1^o$$

$$c^2 = a^2 + b^2$$
$$c^2 = (1.26\text{e-}22)^2 + (4.80\text{e-}23)^2$$
$$c = 1.34765646229\text{e-}22$$

$$p = mv$$
$$v = \frac{p}{m}$$
$$v = \frac{1.35\text{e-}22}{9.11\text{e-}31}$$
$$v = 147\,931\,554 = 1.48\text{e}8\, m/s$$

So the electron is moving at 1.48e8 m/s [69.1° below the path of the original x-ray].

Homework
p723 #1,2