Lesson 9: Coulomb's Law

Charles Augustin de Coulomb


Illustration 1: Charles Coulomb

Before getting into all the hardcore physics that surrounds him, it's a good idea to understand a little about Coulomb.

- He was born in 1736 in Angoulême, France.
- He received the majority of his higher education at the Ecole du Genie at Mezieres (a french military university with a very high reputation, similar to universities like Oxford, Harvard, etc.) from which he graduated in 1761.
- He then spent some time serving as a military engineer in the West Indies and other French outposts, until 1781 when he was permanently stationed in Paris and was able to devote more time to scientific research.

Between 1785-91 he published seven memoirs (papers) on physics.

- One of them, published in 1785, discussed the **inverse square law** of forces between two charged particles. This just means that as you move charges apart, the force between them starts to decrease faster and faster (exponentially).
- In a later memoir he showed that the force is also proportional to the product of the charges, a relationship now called "**Coulomb's Law**".
- For his work, the unit of electrical charge is named after him. This is interesting in that Coulomb was one of the first people to help create the metric system.
- He died in 1806.

The Torsion Balance

When Coulomb was doing his original experiments he decided to use a **torsion balance** to measure the forces between charges.

- You already learned about a torsion balance in Physics 20 when you discussed Henry Cavendish's experiment to measure the value of "G" , the universal gravitational constant.
  - Review Cavendish's work in the Physics 20 notes (Chapter 4 Lesson 29: Newton's Law of Universal Law of Gravitation) if you need to.
- Coulomb was actually doing his experiments about 10 years ***before*** Cavendish.
- He set up his apparatus as shown in Illustration 2 with all spheres charged to have the same sign.
  - He charged one of the free moving spheres by touching it to an already charged object (charging by conduction).
  - He then touched that one sphere to the other free moving sphere (charging *it* by conduction).
  - Each of the free moving spheres was then touched to one of the spheres on the rod (guess what... charging by conduction!).

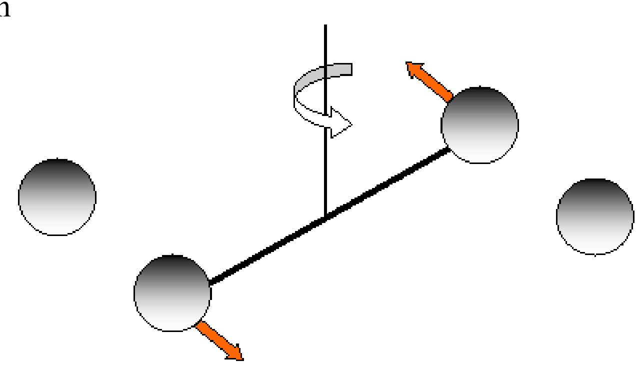


Illustration 2: The Torsion Balance

- Although he didn't know the actual charge on any particular sphere, Coulomb did know that each sphere had an equal charge to all the rest.
    - Coulomb also altered the experiment by using spheres of different sizes so he could get the amounts of charge in different ratios, and by touching spheres to other objects to get other ratios of charges.

Because like charges repel, the spheres on the rod twist away from the other spheres.

- By knowing the distance between the spheres, the force needed to twist them (the torque in the string holding up the rod, from which the torsion balance gets it name), and the charges on the spheres, he could figure out a formula.

In the end, the formula Coulomb finally came up with could be used to calculate the force between any two charges separated by a distance…

$$|F_e| = \frac{k\,q_1 q_2}{r^2}$$

F_e = Force (N)
q = Charge (C)
r = distance between the charges (m)
k = 8.99e9 Nm^2/C^2

- We will calculate the **absolute value** of F_e to get just the **magnitude** of the force.
- Then, we can use information about the charge on the objects to figure out if they are attracting or repelling, and from that we can figure out which direction the force is acting.
    - For example, if both the charges are positive, then we know that they will repel each other by pushing away in opposites directions.
- The reason we need to be so careful with this is that you are used to forces having positive and negative values because **force** is a **vector** with directions like negative meaning "to the left."
    - We are now dealing with a formula where the positive and negative signs come from the **charges**, which are **scalar**.

Special AP Physics Note
In Coulomb's law "k" is sometimes shown as being equal to another set of variables...

$$k = \frac{1}{4\pi\epsilon_o}$$

The symbol ε_o ("epsilon") is a constant known as the vacuum permittivity (aka the permittivity of free space), where ε_o = 8.85e-12 $C^2/N \cdot m^2$

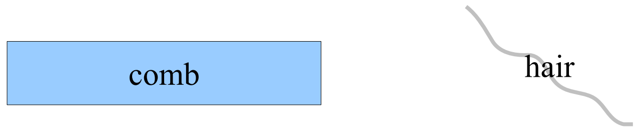
Example 1: A comb with -2.0 μC of charge is 0.15m to the left from a hair with 3.0 μC of charge. **Determine** the force the hair exerts on the comb.

$$F_e = \frac{k\,q_1 q_2}{r^2}$$

$$F_e = \frac{8.99e9(2.0e\text{-}6)(3.0e\text{-}6)}{0.15^2}$$

$$F_e = 2.397333 = 2.4\,N$$

That's enough force to probably make the hair jump towards the comb. Notice that I did not put in the negative sign on the charge of the hair. Now that we know the magnitude of the force, we can decide how that force is acting on the comb.



- Since the comb is negative and the hair is positive, they will attract each other.
- That just means the comb and hair are pulling each other closer together.
- If they are pulling closer together, then the comb is being pulled to the right, and the hair is being pulled to the left (in the diagram shown above).
- We only care about the comb, so we can just say that the force is **2.4 N [right]**.
- Notice that if you had put the negative sign in the formula, your answer would have been negative, and you may have incorrectly said the force was to the left.

You might have noticed the charges used in the last example were micro Coulombs.

- In most lab work you would do at school, or even just in everyday life, charges are usually in this range of about 10^{-6}C (1 μC).
- Only really special cases have charge of 1 C or 2 C... things like a lightning bolt!
- Common subatomic particles can have a charge, as shown in the following table.

Particle	***Charge***
Electron (e^-)	- 1.60e-19 C
Proton (p^+)	+1.60e-19 C
Neutron (n^0)	0 C

- A charge of 1.60e-19 is so important, that it is called an **elementary charge**, and its symbol is just the letter "**e**".
  - This is not "e" for electron, since there is no negative sign on the symbol.
  - If it was written as e^- with the little minus sign on it, then it would refer to an electron.
  - You will find the value of the elementary charge on your data sheet.
- That means that if something has a charge of -1 C, it has a LOT of electrons...

$$1\,C\ x\ \frac{1\,electron}{1.60\text{e-}19\,C} = 6.25\text{e}18\,electrons$$

- Although day to day objects can have this (or more!) electrons, keep in mind that they will often have an equal number of protons to cancel out the charges, for a net charge of zero.

Comparing Electrostatic Force to Gravitational Force

You might have noticed that Coulomb's Law looks almost identical to the formula for Universal Gravitation…

$$F_g = \frac{G m_1 m_2}{r^2}$$

> Both of these formulas are examples of "inverse square laws," formulas where as the distance (r) increases, the measured value exponentially decreases.

- Both formulas calculate a force by multiplying a constant by a measured value of the two objects, divided by the square of the distance separating them.
  There is one significant difference between the two forces.
  - **The gravitational constant G is very small**.
  - **The electrostatic constant k is huge**.

- Because of this difference, **gravitational forces are very weak**, while **electrostatic forces are very strong**.
  - You might disagree with this, thinking about how gravity seems so strong while keeping you stuck to the Earth right now.
  - Think of it this way. When you use a rubbed ebonite rod to attract some bits of paper and lift them up, you are using a whimpy little charged rod's electrostatic force to beat the entire gravitational force of the whole planet Earth pulling down.

- Another important difference is that **gravitational force** can only cause **attraction**, but **electrostatic force** can cause **attraction** *or* **repulsion**.

Multiple Charges in One Dimension (Linear)

Things get a bit more interesting when you start to consider questions that have more than two charges.

- You will almost always deal with three charges in these linear problems.
- In the following example you have three charges lined up and are asked to calculate the net force acting on one of them.
- Do one step at a time, and then combine the answers at the end.

Example 2: The following three charges are arranged as shown. **Determine** the net force acting on the charge on the far right (q_3).

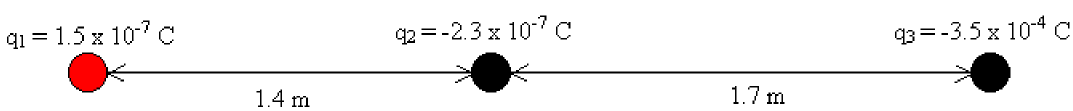


Illustration 3: Arrangement of charges in Example 2

Calculate the force between one pair of charges, then the next pair of charges, and so on until you have calculated all the possible combinations for that particular question. Remember, if you've calculated the force of q_1 on q_2, then you also know the force of q_2 on q_1 ... they're the same!

Step 1: Calculate the force that charge 1 exerts on charge 3...

It does **NOT** matter that there is another charge in between these two… ignore it! It will not effect the calculations that we are doing for these two. Notice that the **total** distance between q_1 and q_3 is 3.1 m , since we need to add 1.4 m and 1.7 m .

$$F_e=\frac{k\,q_1 q_3}{r^2}=\frac{8.99e9(1.5e\text{-}7)(3.5e\text{-}4)}{3.1^2}=0.049112903\,N$$

Since q_1 is positive and q_3 is negative, there will be a force of attraction between them. We know that q_1 is pulling charge q_3 left, while charge q_3 is pulling q_1 to the right. Since all we care about is what is happening to q_3 , all I really need to know from this is that q_3 feels a pull towards the left of 0.049112903 N.

Step 2: Calculate the force that charge 2 exerts on charge 3...

Same thing as above, only now we are dealing with two negative charges, so the force will be repulsive.

$$F_e=\frac{k\,q_2 q_3}{r^2}=\frac{8.99e9(2.3e\text{-}7)(3.5e\text{-}4)}{1.7^2}=0.250413495\,N$$

Since we know that the force is repulsive between these two charges, q_2 is pushing q_3 to the right with a force of 0.250413495 N. Again, we only care about what is happening to q_3.

Step 3: Add you values to find the net force.

- We now need to add the two values from above, being careful about directions. Everything has to be based on the directions of the forces acting on q_3... we don't care about the other charges anymore.
  - The 4.9e-2 N force is pulling q_3 to the left, which is the direction we usually call negative, so we'll put the negative sign on it. **F_e = - 0.049112903 N.**
  - We also have a 2.5e-1 N force pushing to the right. We usually call a vector pointing right positive, so we'll do that here also. **F_e = + 0.250413495 N**

F_{NET} = - 0.049112903 N + +0.250413495 N = 0.201300592 = 0.20 N

- Since the answer is positive, we know that the net force acting on q_3 is **0.20 N [right]**.

Multiple Charges in 2 Dimensions

Doing questions with charges in multiple dimensions are the same as the question you did above. You just need to be careful about directions and use vectors to figure out the problem.

- Figure out all the individual forces between pairs of charges (just like in the 1-D problem).
- Then pay attention to the directions of the forces and calculate the net force as you would any vector problem.
  - This will usually (but not necessarily always) involve a triangle diagram.

Example 3: Three charges are arranged in a right angle triangle as the following diagram shows. **Determine** the force on q_2 .

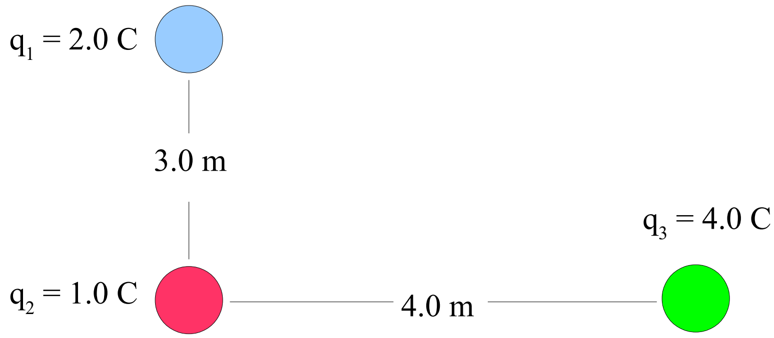


Illustration 4: Arrangement of charges for Example 3

We need to start by calculating the individual forces on q_2 by each of the other charges. These must be calculated individually.

$$_{1}F_{2}=\frac{k\,q_1 q_2}{r^2}=\frac{8.99e9\,x\,2.0\,x\,1.0}{3.0^2}=1\,997\,777\,777\,N$$

$$_{2}F_{3}=\frac{k\,q_2 q_3}{r^2}=\frac{8.99e9\,x\,1.0\,x\,4.0}{4.0^2}=2\,247\,500\,000\,N$$

All of the charges are positive, so all of the forces are repulsive. That means that $_{1}F_{2}$ is a force that is pushing q_2 down, and $_{2}F_{3}$ is a force pushing q_2 to the left.

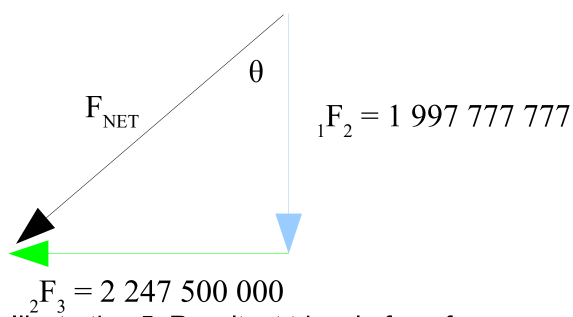


Illustration 5: Resultant triangle from forces acting on the charge.

It's easy enough to calculate F_{NET} using Pythagoras, and figure out the angle using trig.

$c^2 = a^2 + b^2$

$= (1\,997\,777\,777)^2 + (2\,247\,500\,000)^2$

$c = 3\,007\,053\,757 = 3.0e9$ N

$$\tan\theta=\frac{opp}{adj}$$

$$\tan\theta=\frac{2247500000}{1997777777}$$

$$\theta=48.36646=48^{O}$$

The final answer is that q_2 feels a net force of 3.0e9 N at an angle 48^0 clockwise from a vertical line pointing down. Including a diagram with your answer is a great way to show what your direction means.

Homework

1D Questions

p530 #1
p531 #1
p532 #1
p533 #1

2D Question

p534 #2
p535 #1

Review

p540 #5, 6, 12, 17, 23, 25, 26