Lesson 42c: PV Diagrams

From the last section, you were probably wondering what happens when we do something like add heat to a sealed cylinder.
- This sounds like a pretty dangerous idea if you think back to the WHMIS training you had about compressed gas cylinders in Science 10. You might be thinking that it could make the cylinder explode because of an increase in pressure.
- These are the sorts of situations we can consider when we look at **PV diagrams**.
    - A **PV diagram** is a graph of **Pressure** as a function of **Volume**.

> **Pressure** is measured in either Pascals (Pa) or Newtons per metre squared (N/m^2).
>
> **Volume** is measured in metres cubed (m^3).

There are four different situations that you can expect to see shown in PV diagrams:
1. **Isobaric**: the gas is held at a constant pressure
2. **Isochoric**: the gas is held at a constant volume
3. **Isothermal**: the gas is held at a constant temperature
4. **Adiabatic**: No heat flows in or out of the gas

Isobaric

Isobaric is actually a pretty easy thing to show on a PV diagram, and also pretty easy to understand from an example of a situation in which it is happening.
- Imagine that you have a cylinder filled with a gas. At the top is a movable piston that has a few kilograms mass.
    - At the start the pressure is a certain amount for the volume of gas involved.
- We place a heat source under the cylinder and slowly heat it up.
    - Since this is an isobaric process, ***the pressure must stay the same as it was at the start***.
    - We also expect that with the extra internal energy the molecules have gained from the heat source, they will be bouncing around a bit more.
        - This must result in the gas expanding in the cylinder!
- The gas will do work against the piston as it expands, where W = Fd.
    - The force is caused by the pressure acting on the surface area of the piston, so we can use the formula...
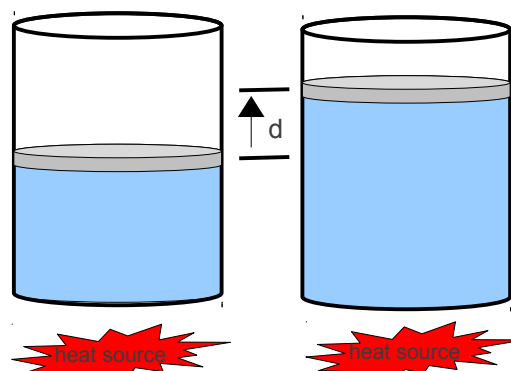
Illustration 1: A cylinder of gas is heated so that it expands while keeping its pressure constant.

$$F = PA$$

F = force (N)
P = pressure (Pa = N/m^2)
A = area of the piston (m^2)

- We can substitute the two formulas together to get...

$$W = Fd \qquad F = PA$$

$$W = PAd$$

$$W = -P\Delta V$$

> Remember from math that volume can be calculated for a cylinder by taking the area of its end and multiplying it by the height. The distance the piston moved up in this example is like a change in height. This means that $\Delta V = Ad$.
> The negative sign is added in the final formula on the AP data sheet so we get the correct sign for W.

- This formula would let us calculate the work done by the gas on the piston as it expands.

Example 1: A gas at 4.0e5 Pa is in a cylinder with a piston. During an isobaric process the gas is heated and expands from 0.25 m³ to 0.55 m³. **Determine** the work done by the gas on the cylinder, and **sketch** a PV diagram of the situation.

$$W = -P\Delta V$$
$$W = -P(V_f - V_i)$$
$$W = -4.0e5\,(0.55 - 0.25)$$
$$W = -1.2e5\ \text{J}$$

The PV diagram shows a gas going from a smaller to a bigger volume, **while the pressure stays constant**.
- Since the volume of the gas increased, we know work was done by the gas.
- The arrow has been added to show the way the process went from start to finish.
- If you look at how we calculated the work done, you might notice that it would have been the same answer if we had calculated the area under the graph. *The area under the line on a PV diagram is the work done!*
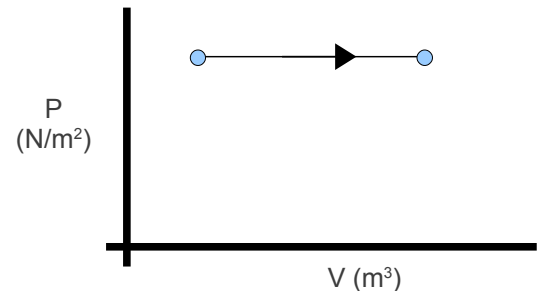 ◦ This is true for any PV diagram, so keep it in mind.

Illustration 2: An isobaric process where the gas does work.

We can also realize something about the temperature change that must be happening if we use the ideal gas law.
- Since the pressure (P), the amount of gas (n), and the gas constant (R) all stay constant...

$$PV = nRT$$
$$V \propto T$$

- Volume (V) is directly related to temperature (T), so as the volume got bigger the temperature was also increasing.
  ◦ "Duh," you say, "we had already said we were heating the cylinder." The point is that the situation agrees with the physics.
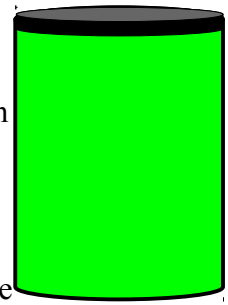
Isochoric

Let's take the previous idea of a gas in a cylinder and just change it a little bit.
In an isochoric process the gas must keep a constant volume, so we will put the gas in a sealed container that can *not* change size or shape.
- As we heat it up more and more, the gas will want to expand even though it can't.
- This will result in an increase of pressure inside the cylinder.
  - The increase in pressure will also create more force against the sides of the cylinder, but since nothing is moving, **no work is done**.
$$W = Fd = F\,(0) = 0 \text{ J}$$

  - The first law of thermodynamics shows that since no work is being done ($W = 0$ J), all that heat being added just increases the internal energy.
$$\Delta U = Q + W$$
$$\Delta U = Q$$

Illustration 3: Volume stays constant during an isochoric process.

For the PV diagram of the isochoric process we've been talking about, we get something like the diagram shown here.
- Notice how the volume stays the same while the pressure builds.
- Since it is a vertical line, there is no area under the line, which means no work is done (just like we figured out a moment ago).

P
(N/m²)

V (m³)
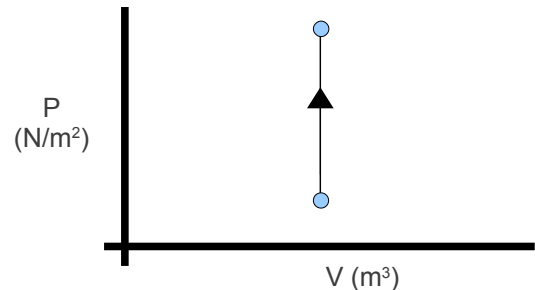
Illustration 4: An isochoric process where no work is done.

We can look at isochoric processes using the ideal gas law also.
- Now the volume (V) stays constant, along with the amount of gas (n), and the gas constant (R)...

$$PV = nRT$$
$$P \; \alpha \; T$$

- This shows us that the pressure is directly related to the temperature. So, as the temperature increased while we heated the cylinder, the pressure increased as well.

Isothermal

Isothermal examples are quite different from the last two, since the **temperature remains constant**. For the example we'll discuss here, we'll try to keep as much of the other ideas the same as possible.
- Imagine a cylinder with a piston on top that is filled with a gas.
- The cylinder is sitting inside a large container filled with water.
  - If anything happens to the cylinder that would normally cause a temperature change in the gas, the water will either absorb or release heat to keep the gas in the cylinder at a constant temperature.
- We have been holding down on the piston a bit, but now we release it so that the piston rises.
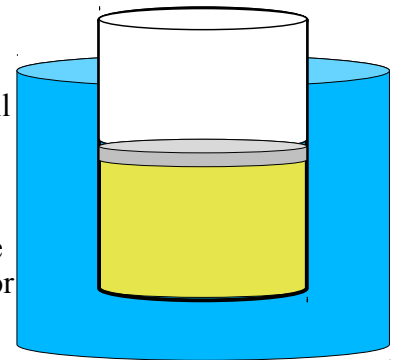
Illustration 5: An isothermal process with a cylinder in a large container of water.

- ◦ The volume of the gas is expanding, and the pressure inside will be dropping.
  - ▪ This is quite different from the other two processes, where one of these stayed constant.
- ◦ Work is being done by the gas (work is negative), so to keep the internal energy (and temperature) constant, the gas must be absorbing heat from the water.
  - ▪ As long as the energy lost (by the gas doing work) equals the heat gained from the water, the internal energy will not change and the temperature stays the same.

$$\Delta U = Q + W$$
$$\Delta U = -W + W$$
$$\Delta U = 0$$

… but Q = - W so...

The PV diagram we get shows a curved line as the pressure and volume both change.
- • The line is called an **isotherm**, because the **temperature stayed the same**.
- • The isotherm is a curve. This happens because when the piston was being released there was a sudden drop in pressure as the gas expanded. As the process continued the pressure dropped more slowly as the volume increased.
- • The area under the curve is still the work done.
  - ◦ Since it is a curve, the only way to accurately find the area is to use calculus. You are not required to know how to do this. Just state "the area under the line is the work done" and you'll be fine.
- • The internal energy change of zero (ΔU = 0) will be true as long as we start and finish on the **same** isotherm on the PV diagram.
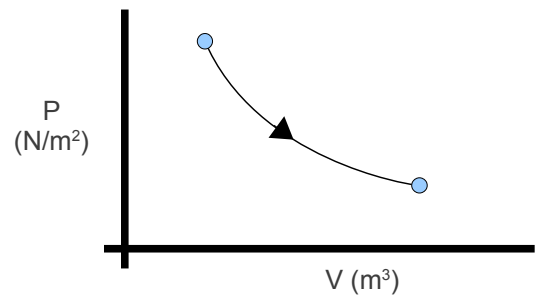

Illustration 6: During an isothermal process the pressure and volume both change, while the temperature stays the same.

Adiabatic

During an adiabatic process no heat is added to the system.
- • This is often confused with isothermal processes, since it is assumed that no heat exchange means the temperature stays constant. ***This is not true!!!***
  - ◦ Think of the isothermal process we just looked at. If we did *not* allow heat to enter from the water the temperature *would* have changed (it would have decreased).
- • During an adiabatic process the heat transfer is zero, so Q = 0 J. This could happen in a situation where the cylinder is surrounded by an insulating material.
- • According to the first law of thermodynamics, this means...

$$\Delta U = Q + W$$
$$\Delta U = 0 + W$$
$$\Delta U = W$$
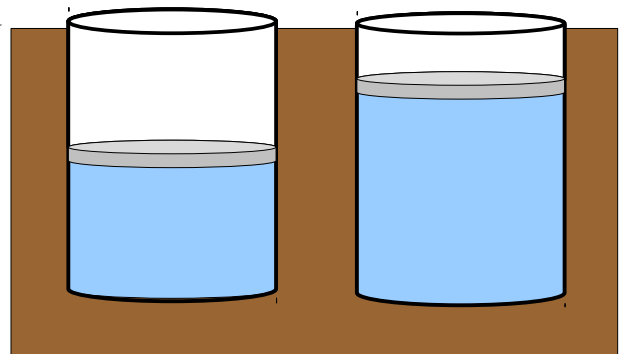
Illustration 7: An adiabatic process happens in an insulating material so heat can not be transfered.

so the change in the internal energy is equal to the work done on or by the gas.
- ◦ If the gas is compressed (work done *on* the gas is positive) the internal energy increases.
- ◦ If the gas expands (work done by the gas is negative) the internal energy decreases.

The PV diagram for an adiabatic process show a special result.

- An adiabatic process looks very much like an isothermal process, but it drops off to a lower point.
  - This means it is on a different isotherm.
    - Remember, isotherms are just lines that show where the temperature stays constant.
    - As we've already discussed, in the adiabatic process where a gas expands, the work done by the gas causes the internal energy to decrease, so the temperature must decrease as well.
      - This is why we drop to a lower isotherm.
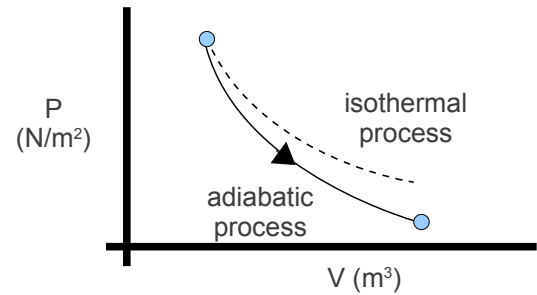
Illustration 8: During an adiabatic process the temperature changes, so we end up on a different isotherm.

We can use a formula for calculating the internal energy of a system based on its temperature to do many problems, but it is especially helpful in adiabatic processes.

- Since $\Delta U = W$ for an adiabatic process, if we figure out the initial and final internal energies, we can figure out the work done on or by the gas without having to use calculus to figure out the area under the PV curve.
- The internal energy is calculated using the formula

$$U = \frac{3}{2}nRT$$

U = internal energy (J)
n = number of moles of gas (mol)
R = gas constant
T = temperature (K)

Example 2: A well insulated cylinder with a piston contains 3.00 mol of gas at 525 K. If it goes through adiabatic expansion and ends at a temperature of 300 K, **determine** how much work the gas did.

The PV diagram we would get for this adiabatic process is shown here. Notice how we started on the 525 K isotherm, but ended on the 300 K isotherm. The temperature did not stay constant, so this process must be adiabatic.
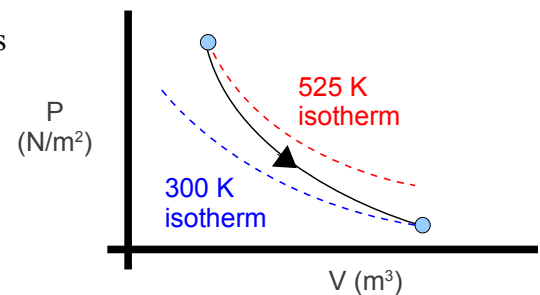
First we calculate the initial internal energy of the gas...

$$U_i = \frac{3}{2}nRT$$
$$U_i = 1.5(3.00)(8.31)(525)$$
$$U_i = 19632.375 \, J$$

…and then the final internal energy...

$$U_f = \frac{3}{2}nRT$$
$$U_f = 1.5(3.00)(8.31)(300)$$
$$U_f = 11218.5 \, J$$

Now we can use $\Delta U = W$ to figure out how much work the gas did...

$$\Delta U = U_f - U_i$$
$$\Delta U = 11218.5 - 19632.375$$
$$\Delta U = W = -8413.875 = -8.41\text{e}3 \, J$$

This means that the gas did 8.41e3 J of work to expand.

Summary Of Processes

Process	Constant	PV Diagram	Ideal Gas Law	First Law of Thermodynamics
Isobaric	Pressure	Horizontal line	V α T	ΔU = Q + W
Isochoric	Volume	Vertical line	P α T	ΔU = Q
Isothermal	Temperature	Curved line	PV α T	ΔU = 0
Adiabatic	No heat exchanged	Curved line (jumps to different isotherm)	PV = nRT (only "nR" are constant)	ΔU = W