Lesson 42a: Kinetic Theory of Ideal Gases

The Theory

The kinetic theory of ideal gases is one of those rare things in physics... it makes sense!

- Basically, from the time you started to learn about gases in elementary school, you were learning about the basic postulates (ideas) of the kinetic theory of ideal gases.
- To remember the postulates given here, try to think of a sealed container holding a gas. Pretty much everything that is said here is the sort of stuff that should pop into your head.

1. The container holds a *huge* number of molecules of the gas.
2. The space between the molecules is *huge* in comparison to the size of the molecules themselves.
3. The individual molecules are in constant, random motion.
4. The molecules can not affect each other at a distance (no gravitational or electrostatic forces on each other).
5. All collisions (molecule-on-molecule or molecule-on-container wall) are **elastic** (***no loss of kinetic energy***).*

* More on this in Physics 30

Root-Mean-Square Speed

Since a container of gas has so many molecules bouncing around all the time (at pretty high velocities!), we can understand why there is pressure against the sides of the container.

- As the gas molecules hit the sides of the container, they will bounce off without losing any of their kinetic energy.
- Because of Newton's 3rd Law, as much as the side of the container exerts a force on the molecule to reflect it inwards, the molecule exerts an equal but opposite force pushing the side outwards.
- The molecules will all be moving at different speeds, so if we talk about a large group of molecules we should talk about the *average* speed.
    - We need to take the average of the speed of the molecules, but the formula for kinetic energy has velocity squared.
    - For this reason we use a special value for the value of the "speed squared" called the **root-mean-square (rms) speed**.
    - The velocity of the gas molecules is found using...

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3k_b T}{\mu}}$$

v_{rms} = root-mean-square speed
R = universal gas constant 8.31 J/(mol•K))
T = Temperature (K)
M = molar mass (kg/mol)
k_b = Boltzmann's Constant (1.38e-23 J/K)
μ = mass of a molecule (kg)

Warning!
Temperatures must always be measured in Kelvin, where...
°C + 273 = K
...and molar mass must be measured in **kg**/mol, not g/mol.

- Notice how the speed is directly related to the square root of the temperature.
  - This makes sense since we assume higher temperature gases have faster moving molecules.
- We also see that the speed is inversely related to the square root of the mass of a molecule.
  - Again logical, since a heavier molecule will move more slowly.

Example 1: **Determine** the rms speed of oxygen molecules (O_2) in a room at 20ºC.

No matter what version of the formula we choose to use, we must first convert the temperature of the air from degrees Celsius to Kelvin...

20ºC + 273 = 293 K

Method 1

If we are going to use the first version of the formula, we need the molar mass (M) of oxygen.

- On the periodic table it shows that oxygen is 16.00 g/mol.
- Oxygen is **diatomic**, so on its own it is always O_2 and has a mass for each molecule of 32.00 g/mol.
- We must change this to kilograms, so divide by 1000 to get M = 0.03200 kg/mol.

Now use the formula...

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$v_{rms} = \sqrt{\frac{3(8.31)(293)}{0.03200}}$$

$$v_{rms} = 478\, m/s$$

Method 2

For this method we need to know the mass of a **single molecule** of oxygen in atomic mass units (u).

- Just like in Method 1, we figure out that oxygen molecules (O_2) have a molar mass of 32.00 g/mol.
- That means that the mass of O_2 is 32.00 u (it's just the same number!).
- The AP data sheet gives the conversion factor to find kilograms, so we'll do that next...

  32.00 u (1.66e-27 kg/u) = 5.312e-26 kg

Warning!
Atomic mass units (u) is a way to measure the mass of very small things. You'll learn more about it in Physics 30. For now, just remember that the molar mass in g/mol is equal to the atomic mass units.

And now we can use the second version of the formula for rms speed...

$$v_{rms} = \sqrt{\frac{3k_b T}{\mu}}$$

$$v_{rms} = \sqrt{\frac{3(1.38e\text{-}23)(293)}{5.312e\text{-}26}}$$

$$v_{rms} = 478\, m/s$$

Average Translational Kinetic Energy

Since all these molecules of gas are moving around so much, it's reasonable to suggest that they have kinetic energy.

- In statistics it is hard to say anything about individuals, but easy to make predictions about large groups.
- That's why we must talk about the average kinetic energy of these molecules, usually referred to as the **average translational kinetic energy**.
  - The "translational" part refers to how the molecule shifts as it moves from place to place.
    - Although this happens in three dimensions, we only focus on the component that directly hits the wall of the container.
  - The formula is based on some really fun relationships between kinetic energy and pressure, and comes out as...

$$K_{avg}=\frac{3}{2}k_b T$$

K_{avg} = average translational kinetic energy (J)
k_b = Boltzmann's Constant (1.38e-23 J/K)
T = Temperature (K)

Example 2: **Determine** the average kinetic energy of oxygen molecules (O_2) in a room at 20°C.

Method 1

If you had just figured out the rms speed of the oxygen molecules like we did in *Example 1*, you could just use the regular kinetic energy formula...

$$E_k=\frac{1}{2}mv^2$$

$$E_k=\frac{1}{2}(5.312\text{e-}26)(478^2)$$

$$E_k=6.07\text{e-}21\ J$$

Method 2

You could also use the formula we just learned...

$$K_{avg}=\frac{3}{2}k_b T$$

$$K_{avg}=\frac{3}{2}(1.38\text{e-}23)(293)$$

$$K_{avg}=6.07\text{e-}21\ J$$

This method reveals that the average kinetic energy of the molecules of a gas does ***not*** depend on the gas in question, only on the temperature.
This means that the temperature of a gas is a direct representation of the average energy of the molecules of that gas.
For example, in a room the heavier gas molecules (like CO_2) will be moving slower, and lighter gas molecules (like O_2) will move faster, and yet have the same average kinetic energy if they share the same temperature.
This also means that the pressure in a container is related to its temperature because of the energy of the individual molecules in the container hitting its sides.