Lesson 60: Bernoulli's Equation (AP Only)

Bernoulli's Equation looks at the pressure at two different locations in a moving fluid.

- It is really intimidating when you first see it, but it's not as bad as it might look.

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

P = pressure at that point (Pa)
ρ = density of fluid (kg/m^3)
g = gravity (m/s^2)
y = vertical height at that point (m)
v = velocity of flow at that point (m/s)

- First of all, it ends up reading out like the conservation of mechanical energy formulas we used a while back.
  - Notice how the second and third terms on each side look a lot like mgh and $\frac{1}{2}mv^2$. That should make remembering them easier.
- Second, when you actually use the formula you will often find that you get to cancel several things on both sides.
- An example is the best way to see how to use the formula.

Example 1: You have a couple of garden hoses connected together to water your back yard. Because you're a physics nerd, you put a gauge where the two hoses connect to each other. The hose ends with your sprinkler. When you put a crimp in the hose to stop water flow, you noticed that the pressure registered 5.00e3 Pa. When the sprinkler is running, what is the velocity of the water flowing out.

We'll assume that the gauge on the hoses is at the same height as the sprinkler (pretty fair bet), so y_1 equals y_2. Since those terms will be identical on both sides, they'll cancel.

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 + \cancel{\rho g y_2} + \frac{1}{2}\rho v_1^2 = P_2 + \cancel{\rho g y_2} + \frac{1}{2}\rho v_2^2$$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

When we measured the initial pressure with the gauge, we crimped the hose, so the velocity of the flow was zero. That cancels another term on the left side.

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 + \frac{1}{2}\rho 0^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 = P_2 + \frac{1}{2}\rho v_2^2$$

The pressure that we measured with the ***gauge*** must be ***gauge pressure***, so the absolute pressure we will use for P_1 is 1 atmosphere plus the 5.00e3 Pa. As far as the pressure at P_2 goes, any time a fluid is exposed to the atmosphere, its pressure must be 1 atmosphere.

Remember...
P_1 = 1 atm + 5.00e3 Pa
P_2 = 1 atm

$$P_1 = P_2 + \frac{1}{2}\rho v_2^2$$
$$(1\,atm + 5.00e3\,Pa) = P_2 + \frac{1}{2}\rho v_2^2$$
$$(\cancel{1\,atm} + 5.00e3\,Pa) = \cancel{1\,atm} + \frac{1}{2}\rho v_2^2$$
$$5.00e3 = \frac{1}{2}\rho v_2^2$$
$$v_2^2 = \frac{2(5.00e3)}{\rho}$$
$$v_2 = \sqrt{\frac{2(5.00e3)}{1.00e3}}$$
$$v_2 = 3.16\,m/s$$