Lesson 59: Principle of Continuity (AP Only)

We've spent a lot of time so far looking at hydrostatics, fluids at rest.

- Even when we looked at problems with moving fluids (like Pascal's Principle), you would not describe the fluid as flowing, like water through pipes in your home. The fluid just shifted around a bit.
- Now we will look at fluids in motion.
  - We will continue to assume that the fluids we deal with are ***incompressible***, so that whatever volume we have stays constant.

Imagine that you are standing in the river from the movie Charlie and the Chocolate Factory.

- As you gobble up the chocolate, you notice that there is a lot rushing past you.
  - Near the banks, where there is more friction, it is probably moving slower.
  - Out in the middle, where there is less friction, it is moving faster.
- Wherever you choose to stand, at one particular location, you will notice that the velocity of the fluid flowing past you is probably pretty constant.
  - As long as this is the case (at that location the flow remains constant), we can say that we have **continuity**.

The **principle of continuity** is sometimes stated by talking about something called **volume flow rate**.

- **Volume flow rate** says that we can measure the volume of fluid that flows past a point each second.

$$Volume\ flow\ rate = \frac{V}{t}$$

$$Volume\ flow\ rate = \frac{Ad}{t}$$

$$Volume\ flow\ rate = A\frac{d}{t}$$

$$Volume\ flow\ rate = Av$$

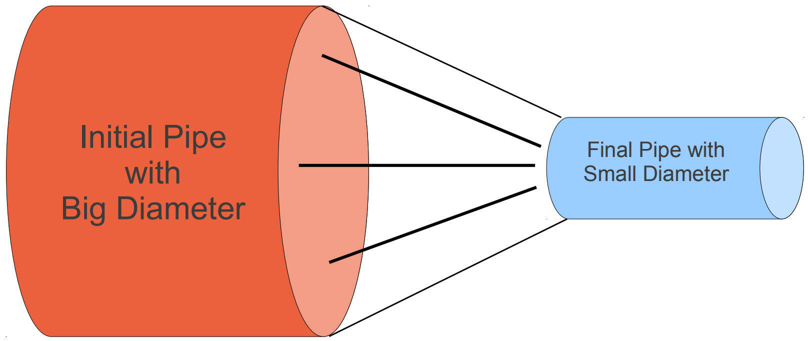
Volume is equal to the cross sectional area of the "river" times how far it has moved. Displacement over time is equal to velocity.

Volume flow rate (m^3/s)
A = area (m^2)
v = velocity (m/s)

- The **principle of continuity** says that the **volume flow rate** is equal along an isolated line in a stream of a fluid.
  - So the fluid must be incompressible, and moving at a constant velocity.

So what happens if we try to put a fluid through a pipe that changes in diameter.

- Come on, admit it. You've all stuck your thumb into the end of a hose so you could spray someone with water and soak them.
  - When you stuck your thumb in the hose, you made the diameter of the hole smaller.
  - The volume flow rate (the amount of water coming out) needs to stay constant, since otherwise the fluid would have to compress (it can't) or the hose would burst (don't see that happening).
- So the volume flow rate at one spot is equal to the volume flow rate at another spot.



$$Volume\ Flow\ Rate_1 = Volume\ Flow\ Rate_2$$
$$A_1 v_1 = A_2 v_2$$

Example 1: You have a hose that has an opening with a 1.70 cm diameter opening. The water is coming out at 3.2 m/s. If you stick your thumb in and reduce the opening to a diameter of 0.40 cm, **determine** the velocity of the water as it exits.

$$A_1 v_1 = A_2 v_2$$
$$v_2 = \frac{A_1 v_1}{A_2}$$
$$v_2 = \frac{\pi r_1^2 \ v_1}{\pi r_2^2}$$
$$v_2 = \frac{r_1^2 \ v_1}{r_2^2}$$
$$v_2 = \frac{0.0085^2 (3.2)}{0.0020^2}$$
$$v_2 = 57.8\, m/s$$

> Remember to divide the diameters that were given by 2 to get the radius for each part.