

3: Torque

Torque Basics

Torque is a concept that is very closely related to the ideas we've been developing about force.

- Just like force is a push or a pull on an object, **torque** is a **twist**.
- **Torque** is the tendency of a force to twist a **rigid object**.
 - We measure the “twist” from a point where the object can pivot, often called the **fulcrum point**.

We will only discuss “rigid” objects in our study of torque. A rigid object will not bend or flex as a force is applied to it.

As an example of what torque means and why it is important, think of the example of a door.

- Usually the door knob is placed far away from the fulcrum (where the door is attached to the wall with hinges) as shown in *Illustration 1*.
 - If you pull on the knob to open the door, you have a pretty easy time because the force you are using causes a lot of torque (twisting) around the fulcrum (hinges) to open the door.
- Imagine that the door knob is now placed near the centre of the door, closer to the hinge, as shown in *Illustration 2*.
 - If you pull on the door knob with the same force as before, it will be more difficult to open the door.
 - This is because the torque (twisting force) has decreased.



Illustration 1: The torque is quite large when the force is applied far away from the fulcrum.



Illustration 2: If the force is applied closer to the fulcrum, the torque is smaller.

The formula to measure the amount of torque is a short one, but applying it depends on the question being asked.

$$\tau = F \ell$$

τ = torque (N·m)

F = force (N)

ℓ = **lever arm distance** (m)

- In the illustrations shown above, the force was kept constant but the **lever arm distance** was changed. This is why we did not get as much torque in the second situation.

DID YOU KNOW?
The symbol τ is the Greek letter tau.

At the end of a calculation of torque, it is common to give the torque a positive or negative value, depending on the direction it is turning.

- We do this the same way we've handled anything rotational in the past couple lessons...
 - **Clockwise (CW) torque = negative**
 - **Counterclockwise (CCW) torque = positive**

Example 1: You are trying to loosen a rusty bolt by using a wrench. You apply 250 N of force to try to turn it counterclockwise. **Determine** the torque if you hold the wrench (a) 3.00 cm and (b) 12.0 cm from the bolt. **Explain** which method is best.

$$\begin{array}{ll} \tau = F \ell & \tau = F \ell \\ \text{a) } \tau = 250(0.0300) & \text{b) } \tau = 250(0.120) \\ \tau = 7.50 \text{ N}\cdot\text{m} & \tau = 30.0 \text{ N}\cdot\text{m} \end{array}$$

Since the question said we are turning it counterclockwise, the torque on the bolt is positive in both answers.

The second method (holding it at 12.0cm) is better, since it maximizes the effect of the force being exerted so that the torque is larger.

Torque if the Force is Applied at an Angle

In some situations a force may be applied at an angle so that the torque is less than it could be.

- In these situations we must draw a **line of action** that points in the same direction as the force, and measure the lever arm distance from the fulcrum to this **line**.

Example 2: A person is pulling on a door with a force of 210 N to open it. The door knob is 72.0 cm from the hinges. **Determine** the difference in torque if the person pulls straight back on the door knob, as opposed to pulling at a 65° angle from the door.

In the first situation the person pulls straight back, so the force is perpendicular to the door and the lever arm length is 0.720 m.

$$\begin{array}{l} \tau = F \ell \\ \tau = 210(0.720) \\ \tau = -151 \text{ N}\cdot\text{m} \end{array}$$

The torque is negative since it is turning clockwise.

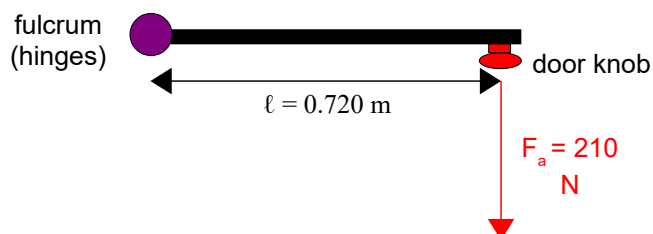


Illustration 3: Pulling straight back on the door knob maximizes the torque.

When the person pulls against the knob at an angle, they are not using the force as well as they were previously to create torque. The distance from the door knob to the hinge is no longer the lever arm distance.

We extend the **line of action** out from the force, and then draw the *new* lever arm distance perpendicular to this line and pointing to the fulcrum. This makes a right angle triangle, so we can use trig to figure out the length of our new lever arm...

$$\begin{array}{l} \sin \theta = \frac{\text{opp}}{\text{hyp}} \\ \text{opp} = \sin \theta (\text{hyp}) \\ \text{opp} = \sin 65^\circ (0.720) \\ \text{opp} = 0.653 \text{ m} \end{array}$$

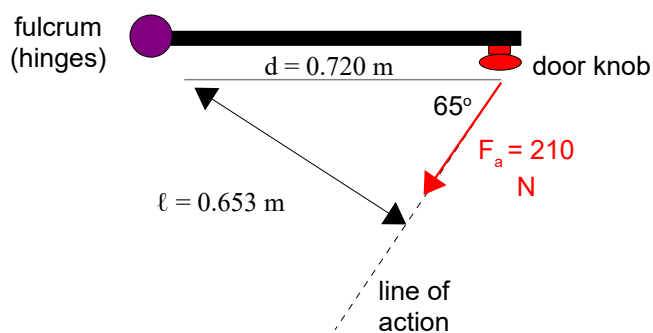


Illustration 4: Pulling on the handle at a 65 degree angle decreases the lever arm distance, so the torque also decreases.

Now we have our special new lever arm and can calculate the torque (negative for clockwise turning) for this situation...

$$\begin{aligned}\tau &= F \ell \\ \tau &= 210(0.653) \\ \tau &= -137 \text{ N}\cdot\text{m}\end{aligned}$$

The percent difference in the torque between the two situations is about 9.72%. That's pretty significant.

Torque for Objects in Equilibrium

We need to look at lots of situations where all the forces acting on an object cancel out ($F_{\text{NET}} = 0$) so that the object is not moving.

- This implies that the net torque acting on the object must also be zero, since otherwise the object would start to rotate.
- We can use this in situations involving rigid objects in equilibrium to predict things about the forces they will experience.

Example 3: A 900N person is walking across a bridge that is 15.0 m long. The bridge itself weighs 3000 N. The person has walked 2.00 m away from one end of the bridge. **Sketch** a diagram of this situation and **determine** the force the support at the opposite end of the bridge need to exert to hold the bridge up at that moment of time.

We will treat the end of the bridge that the person is walking away from as the fulcrum point, and assume that the bridge is trying to rotate clockwise because of the torque being applied. All distances will be measured from the fulcrum. If we can figure out how much the torque is on the bridge, we can figure out the force acting down on the support at the opposite end. The force exerted upwards by the support must be equal to this to be able to keep the bridge in equilibrium.

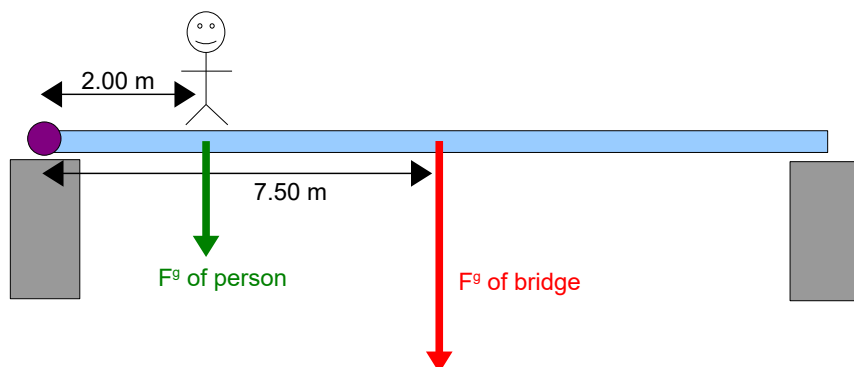


Illustration 5: Person standing on a bridge.

We draw in force vectors to show where the weight of the bridge and the person are. We draw the weight of the bridge exactly in the middle, 7.50 m from the fulcrum (assuming the weight is evenly spread across the two supports). The weight of the person is 2.00 m away from the fulcrum.

Now we calculate the torque due to these two forces, and the total torque acting on the bridge...

Bridge	Person	Total Torque
$\tau_{\text{bridge}} = F \ell$	$\tau_{\text{person}} = F \ell$	$\tau_{\text{total}} = \tau_{\text{bridge}} + \tau_{\text{person}}$
$\tau_{\text{bridge}} = 3000(7.50)$	$\tau_{\text{person}} = 900(2.00)$	$\tau_{\text{total}} = -22500 + -1800$
$\tau_{\text{bridge}} = -22500 \text{ N}\cdot\text{m}$	$\tau_{\text{person}} = -1800 \text{ N}\cdot\text{m}$	$\tau_{\text{total}} = -24300 \text{ N}\cdot\text{m}$

Finally, we need to see how much force this torque causes on a point 15.0 m away from the fulcrum, which is where the far support is located.

$$\tau = F \ell$$

$$F = \frac{\tau}{\ell}$$

$$F = \frac{24300}{15.0}$$

$$F = 1620 \text{ N} = 1.62\text{e}3 \text{ N}$$

We use a positive value for torque here, since the clockwise or counterclockwise torque does not necessarily relate to the up or down direction of the force.

There is a combined force of 1.62e3 N on the support at the far end. This means that the support must be able to push up with this much force.

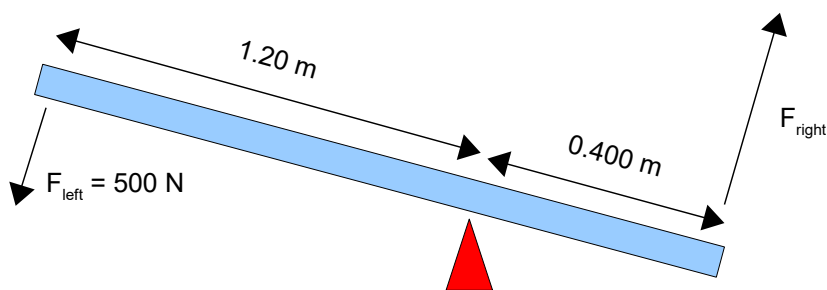
Torque and Levers

You can use the concepts of torque to solve problems involving levers.

- A lever is usually just a rigid rod that has a fulcrum somewhere along it.
- Levers come in three types, depending on where the fulcrum is placed. In some cases this results in an increase in force at some point along the lever, in other cases it is done for convenience.

Example 4: A person is using a lever arranged somewhat like a see-saw to help lift a tree stump out of the ground. The lever is 1.60 m long, and the fulcrum is placed 0.400 m from the tree stump. Assuming that the lever has negligible weight (you don't have to count it), and that all forces are perpendicular to the lever, **determine** the force at the tree stump if the other end is pushed down with a force of 500 N.

A quick sketch of the situation would look something like this...



The force that is being applied on the left side will cause a torque that is equal to the torque on the right side.

$$\begin{aligned}\tau_{\text{left}} &= F_{\text{left}} \ell \\ \tau_{\text{left}} &= 500(1.20) \\ \tau_{\text{left}} &= 600 \text{ N}\cdot\text{m}\end{aligned}$$

$$\begin{aligned}\tau_{\text{right}} &= F_{\text{right}} \ell \\ F_{\text{right}} &= \frac{\tau_{\text{right}}}{\ell} \\ F_{\text{right}} &= \frac{600}{0.400} \\ F_{\text{right}} &= 1500 \text{ N} = 1.50\text{e}3 \text{ N}\end{aligned}$$

So, by having different distances from the fulcrum we can have the advantage of increasing the force exerted on one part of the lever.