2: Angular Acceleration

If we are going to define angular acceleration, it must be similar to what we have already said about linear acceleration…

$$a=\frac{\Delta v}{\Delta t}=\frac{v_f-v_i}{t}$$

- We need to make some adjustments, like using the symbol omega instead of velocity, and switch out to a new symbol for angular acceleration.

$$\alpha=\frac{\Delta \omega}{t}=\frac{\omega_f-\omega_i}{t}$$

α = angular acceleration (rad/s^2 or 1/s^2)
ω_f = final angular velocity (rads/s)
ω_i = initial angular velocity (rads/s)
t = time (s)

Keep in mind that since angular velocity is the same for all points on a rotating body, the angular acceleration must also be constant.

The formula often gets solved for final angular velocity...

$$\omega_f = \omega_i + \alpha t$$

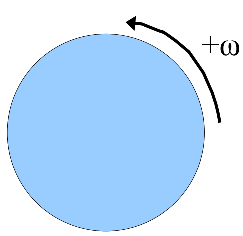
Special Note: The AP data sheet does not like showing *final* and *initial* subscripts. You will see the formula above shown this way on the AP data sheet...

$$\omega = \omega_o + \alpha t$$

The omega with no subscript is *final*, and the one with a little zero subscript (read as “naught”) is the *initial*. You will see this throughout the AP Physics supplied data sheet.

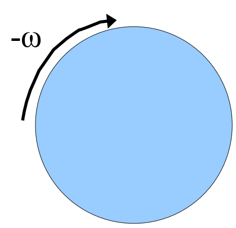
We determine the sign for angular acceleration based on the angular velocity and if it is speeding up or slowing. Remember angular velocity is...

- positive when we are spinning counterclockwise
- negative when we are spinning clockwise.



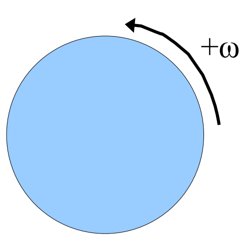
ω is **positive**

if speeding up
α is **positive**



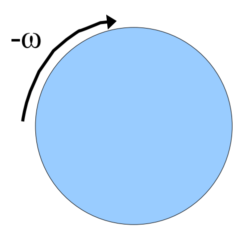
ω is **negative**

if speeding up
α is **negative**



ω is **positive**

if slowing
α is **negative**



ω is **negative**

if slowing
α is **positive**

Example 1: A large industrial fan for cooling a warehouse is switched on. Because it is so large, it takes 2.00 minutes for it to spin up from rest to its final frequency of 3000 rpm [clockwise]. **Determine** its angular acceleration.

Its initial angular velocity is zero.

It has a final frequency of 3000 rpm. We need to calculate a final angular velocity from that.

$$f = 3000\frac{rev}{min}\left(\frac{1\,min}{60\,s}\right) = 50\,Hz \qquad \omega_f = 2\pi f = 2(3.14)(50) = 314\,rad/s[clockwise]$$

This is a fan spinning clockwise, so we will use the angular velocity as a negative value.

$$\alpha = \frac{\Delta\omega}{t} = \frac{\omega - \omega_o}{t} = \frac{-314 - 0}{120} = -2.62\,rad/s^2$$

Relating Angular Acceleration to Linear Acceleration

We can do the same sort of relationship here with acceleration as we did in the last set of notes regarding velocity.

$$a = \frac{\Delta v}{\Delta t} \quad \text{and we know} \quad v = r\,\omega$$

- We can't just throw this into the top of the formula, since although the velocity is changing (the delta symbol), in our formula for velocity only the angular velocity, ω, can be changing. The radius must stay constant.

$$a = \frac{\Delta v}{\Delta t}$$
$$a = \frac{r\,\Delta\omega}{\Delta t}$$
$$a = r\frac{\Delta\omega}{\Delta t}$$

but since $\alpha = \frac{\Delta\omega}{\Delta t}$

$$a = r\,\alpha$$

a = tangential acceleration (m/s^2)
r = radius (m)
α = angular acceleration (rad/s^2)

Formating note: If seeing the letter "a" for tangential acceleration and "alpha" (α) for angular acceleration the same formula makes them hard to tell apart (especially since you probably write your letter "a" as "*a*"), you can write the tangential acceleration as a_T to tell them apart more easily…

$$a_T = r\,\alpha$$

Example 2: If the fan blade in *Example 1* has a radius of 1.5 m, determine the tangential acceleration at the outside edge of the blades.

$$a = r\,\alpha$$
$$a = 1.5\,(-2.61666)$$
$$a = 3.925 = 3.9\ m/s^2$$

We can come up with common analogies for many of the formulas you have studied in linear acceleration in Physics 20 to ones that we can use in angular motion questions.

- Note, the AP data sheet uses x instead of d for displacement.

Alberta Physics 20 Data Sheet Linear Motion	**AP Physics Data Sheet Linear Motion**	**AP Physics Data Sheet Angular Motion**
$v_f = v_i + at$	$v = v_o + at$	$\omega = \omega_o + \alpha t$
$d = \left(\frac{v_f + v_i}{2}\right)t$	$x = x_o + \frac{1}{2}(v_o + v)\,t$	$\theta = \theta_o + \frac{1}{2}(\omega_o + \omega)\,t$
$d = v_i t + \frac{1}{2}at^2$	$x = x_o + v_o t + \frac{1}{2}at^2$	$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$
$v_f^2 = v_i^2 + 2ad$	$v^2 = v_o^2 + 2a(x - x_o)$	$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$

Example 3: To throw a curve ball, a pitcher gives the ball an initial angular velocity of 40.0 rad/s [counterclockwise]. When the catcher catches the ball 0.600s later, air resistance has decreased its angular velocity to 34.0 rad/s [counterclockwise].

a) **Determine** the ball's angular acceleration.

We will treat both the angular velocities as positive, since they are both counterclockwise.

$$\omega = \omega_o + \alpha t$$

$$\alpha = \frac{\omega - \omega_o}{t} = \frac{34.0 - 40.0}{0.600} = \frac{-6.0}{0.600} = -10.0\,rad/s^2$$

b) **Determine** the ball’s the angular velocity 0.500 s after it is thrown.

$$\omega = \omega_o + \alpha t$$
$$= 40.0 + -10.0\,(0.500)$$
$$\omega = 35.0 \text{ rad/s [counterclockwise]}$$

c) **Determine** how many rotations (complete revolutions) the ball makes before being caught.

We will set its original angular position at zero ($\theta_o = 0$)

$$\theta = \theta_o + \frac{1}{2}(\omega_o + \omega)\,t$$
$$= 0 + \frac{1}{2}(40.0 + 34.0)\,0.600$$
$$\theta = 22.2\ rad$$

Remember that 1 complete revolution is 2π rad, so…

$$22.2\,rad\left(\frac{1\,rev}{2\,\pi\,rad}\right)$$

$$22.2\,rad\left(\frac{1\,rev}{2(3.14)\,rad}\right)=3.535=3.54\,rev$$

It made 3.54 rotations.

d) **Determine** the ball's angular velocity after it completes its first full spin.

After completing one full spin, starting from $\theta_o = 0$, it's angular position is $\theta = 2\pi$ rad.

$$\omega^2 = \omega_o^2 + 2\alpha\,(\theta - \theta_o)$$
$$= 40.0^2 + 2(-10.0)\,(2(3.14) - 0)$$
$$\omega^2 = 1474.4$$
$$\omega = 38.4\ rad/s$$