Lesson 5: Pressure in Static Fluid Columns

Imagine a submarine that is about to dive under the ocean.

- Even before it dives, it has about 100 kPa of atmospheric pressure acting down on it, since there is the weight of about 100 km of air above it pressing down.
- Most people get the basic idea that as you go underwater, the pressure increases. There is now the additional pressure of the water above the submarine pressing down on it.
- Think of the air and the water as like columns of fluid above the submarine pushing down on it.

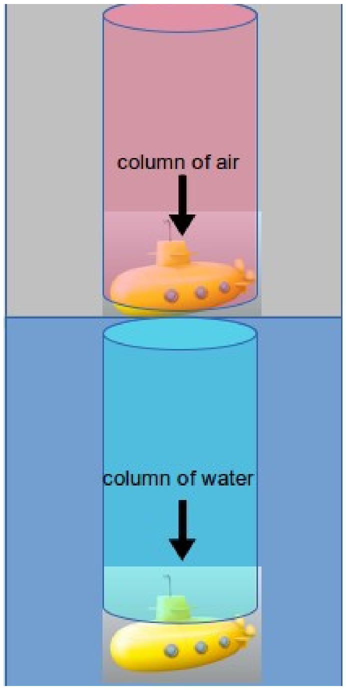

Illustration 1: Columns of fluid above the submarine.

At the surface of the water, the submarine will be experiencing 1 atm (about 1.0e5 Pa) of pressure just from the normal atmosphere above it.

- Now if it goes underwater, you need to consider how the weight of the water above the submarine increases that pressure.
- It makes sense, since pressure is just a force acting over an area. In this case, the additional force of the weight of the water is pushing down across the area of the submarine.

In Lesson 54, recall how we measured the weight of something if we used measurements of density and volume.

$$F_g = mg \longleftrightarrow \rho = \frac{m}{V}$$
$$F_g = mg \longleftrightarrow m = \rho V$$
$$F_g = \rho V g$$

- The difference is that now we are going to need to do the measurement based on pressure.

$$P = \frac{F}{A} \leftrightarrow F_g = \rho V g$$
$$P = \frac{\rho V g}{A}$$
$$P = \frac{\rho A h g}{A}$$
$$P = \rho g h$$

Remember that V = A h . In this case the height is the height of the column of fluid above you.

- So the above formula would tell us how much **pressure there is due to the fluid above us**.
    - We need to remember that there is still the **original pressure from the atmosphere** above us to be added to this additional **pressure from the fluid**.

$$P = P_o + \rho g h$$

P = Final pressure (Pa)
P_o = Initial pressure (Pa)
ρ = density of the fluid (kg/m^3)
g = gravity (m/s^2)
h = height of fluid (m)

- This formula does not just apply to diving underwater (it can be any fluid), and it does not have to start with an initial pressure at the surface of the water (although it often will).

Example 1: My wife goes swimming at Kinsman's deep diving pool. She goes down to a depth of 6.7m. **Determine** the total pressure she is experiencing, and **compare** it to regular atmospheric pressure.

At the surface she would already have experienced 1 atmosphere of regular pressure (that's P_1). We have to add the additional pressure from the water above her (ρgh).

$$P = P_o + \rho g h$$
$$P = 1.00e5 + 1.00e3(9.81)(6.7)$$
$$P = 1.66e5\,Pa$$

This is the total pressure acting on my wife during the dive. It is the sum of the original atmospheric pressure PLUS the additional pressure of the water. This is called the **absolute pressure**.

To compare it to regular atmospheric pressure, let's divide it by one atmosphere and see how many times greater it is...

$$ratio = \frac{1.66e5}{1.00e5} = 1.66$$

We could also subtract the original air pressure from our total absolute pressure to say how much pressure there is from the water alone...

$$1.66e5 - 1.00e5 = 6.6e4\,Pa$$

...or we could calculate the pressure from the water alone using

$$P = \rho g h$$
$$P = 1.00e3(9.81)(6.7)$$
$$P = 65\,727 = 6.6e4\,Pa$$

Measured this way we are ignoring the original air pressure that's always acting on her (we don't really notice it day to day anyways, right?). This is called the **gauge pressure**. Think of it like using a tire pressure gauge, which would show zero until you measure the pressure of the tire; it only measures the additional pressure of the tire.

Example 2: The Mariana Trench is the deepest point of the world's oceans. It reaches an estimated depth of 10 971 m. **Determine** the gauge and absolute pressures at this depth in atmospheres.

To figure out the gauge pressure we only want the additional pressure of the water alone...

$$P = \rho g h$$
$$P = 1.00e3(9.81)(10971)$$
$$P = 107\,625\,510 = 1.08e8\,Pa$$

To measure the absolute pressure, we could just add one atmosphere to the above answer. I'm still going to show the full calculation, pretending we didn't do the first calculation.

$$P = P_o + \rho g h$$
$$P = 1.00e5 + 1.00e3(9.81)(10971)$$
$$P = 107\,725\,510 = 1.08e8\,Pa$$

Umm, did you notice that after rounding the two answers are the same? That's just because at such an incredible depth the pressure of the water alone is much greater than the puny effect of the Earth's atmosphere.