

## Lesson 4: Pascal's Principle

Imagine that you have a container of fluid.

- From what we've learned so far, we know that the pressure the fluid exerts on the sides of the container is the same everywhere.
  - If they were not, the fluid would no longer be static.
- Now we exert a force somewhere on the outside of the container. What can we expect will happen to the pressure inside the container?
  - Well, for starters, it makes sense to say that the pressure will increase.
  - The important part is that, according to **Pascal's Principle**, the pressure will increase everywhere in the fluid, not just where you are applying the force.

### Application of Pascal's Principle

Probably the most easily understood example of Pascal's principle at work is when a hydraulic lift (like in a mechanics garage) lifts up a large mass.

- On one side we have a **small cylinder** (filled with an incompressible liquid) with a piston. This is where we will exert a force downwards.

$$\Delta P = \frac{F_1}{A_1}$$

We don't care what the original pressure was in the cylinder, only that the force we are exerting now causes a change in pressure.

- This is connected by a pipe to another, **larger cylinder**, with a large piston, where the large mass will be lifted by a force upwards.

$$\Delta P = \frac{F_2}{A_2}$$

According to Pascal's principle, the change on both sides must be the same.

- If  $\Delta P$  is the same, then we can combine the two formulas from above into one equation...

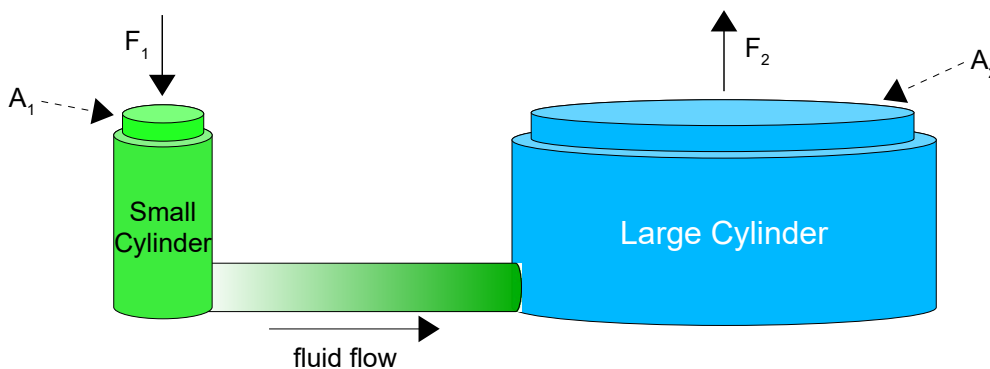


Illustration 1: A hydraulic lift works according to Pascal's principle.

$$\Delta P = \Delta P$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

- Since  $A_2 > A_1$ , then  $F_2 > F_1$  for the two sides to be equal. This just means that we can exert a small force on the small piston and get a bigger force on the big piston.

Does this sound like getting something for nothing? It isn't, if you keep a few ideas in mind.

- First of all, even though we only push the small piston with a small force, we have to push it a big distance down. On the other side the large piston has a large force up, but it only moves a small distance up.
  - This is because we are moving a constant volume of fluid (it is incompressible, remember). This can be calculated for either side...

$$V = A_1 d_1 \quad V = A_2 d_2$$

$$A_1 d_1 = A_2 d_2$$

- Both of these sides are equal, so we can multiply a formula by either side and not change anything...

$$\begin{aligned} \frac{F_1}{A_1} &= \frac{F_2}{A_2} \\ \frac{F_1}{A_1}(A_1 d_1) &= \frac{F_2}{A_2}(A_2 d_2) \\ F_1 d_1 &= F_2 d_2 \\ W_1 &= W_2 \end{aligned}$$

If I have a formula like  $5x = 10$  I can multiply both sides by 2 and get  $10x = 20$  and not really change anything. Same idea here; from above we know that  $Ad$  calculated using either pair of numbers is the same, so if I multiply using  $A_1 d_1$  or  $A_2 d_2$ , it's really the same thing.

- This shows that the work done is the same on both pistons. Conservation of energy is ok, since we haven't created or destroyed any energy.

**Example 1:** I want to build a hydraulic press to be able to squeeze all my gold bars down to thin gold disks. The small piston has an radius of 1.0 cm and I will be able to exert a force of 150 N on it. If the large piston has a radius of 10 cm **determine** the force against the gold bars.

$$\begin{aligned} \frac{F_1}{A_1} &= \frac{F_2}{A_2} \\ \frac{F_1}{\pi r_1^2} &= \frac{F_2}{\pi r_2^2} \\ \frac{F_1}{r_1^2} &= \frac{F_2}{r_2^2} \\ \frac{150}{0.010^2} &= \frac{F_2}{0.10^2} \\ F_2 &= 1.5 \times 10^4 \text{ N} \end{aligned}$$

We can cancel pi on both sides.

## Homework

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