Lesson 50: Resonating Air Columns

Many musical instruments depend on the musician in some way moving air through the instrument.

- This includes brass and woodwind instruments, as well as instruments like pipe organs.
- The air that is moving through these columns (also called *pipes* or *tubes*) resonates as it passes through, creating the sound of the notes we hear.
  - The column can be any tube, even if it has been bent into different shapes or has holes cut into it.
- All instruments like this can be divided into two categories, **open ended** or **closed ended**.

An **open ended** instrument has both ends open to the air.

- An example would be an instrument like a trumpet. You blow in through one end and the sound comes out the other end of the pipe.
- The keys on the trumpet allow the air to move through the "pipe" in different ways so that different notes can be played.

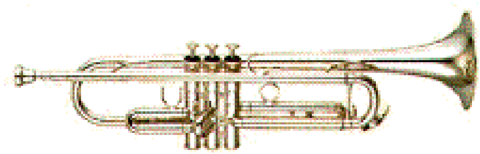

Illustration 1: A trumpet is open ended.

Illustration 2: A flute is closed ended.

A **closed ended** instrument has one end closed off, and the other end open.

- An example would be an instrument like some organ pipes (although in some designs they are open), or a flute.
- Although you blow in through the mouth piece of a flute, the opening you're blowing into isn't at the end of the column, it's along the side of the flute. The end of the column is closed off near the mouth piece.

The frequencies of sounds made by these two types of instruments are different because of the different ways that air will move at a **closed** or **open** end of the column.

- The diagrams that I will be drawing are based on the way the air will move as a wave.
- Although the sound waves actually travel through the columns as longitudinal waves, I will be drawing transverse waves. This is just because they are easier to draw and recognize in the diagrams.

Identifying Fractions of Wavelengths

Before we look at the diagrams of the columns, let's make sure that you know what fractions of a wave look like. This will be important in the way you interpret the diagrams later.

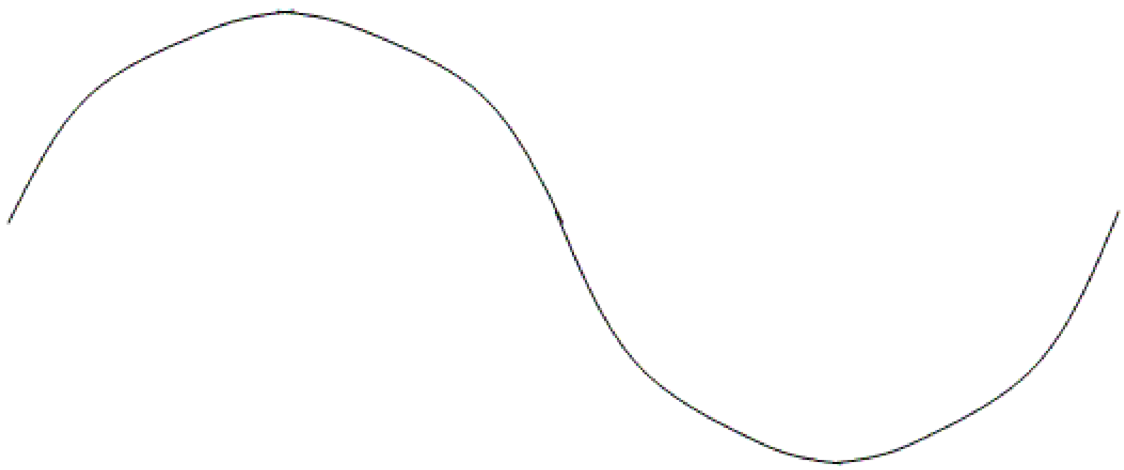

Illustration 3: One wavelength (λ).

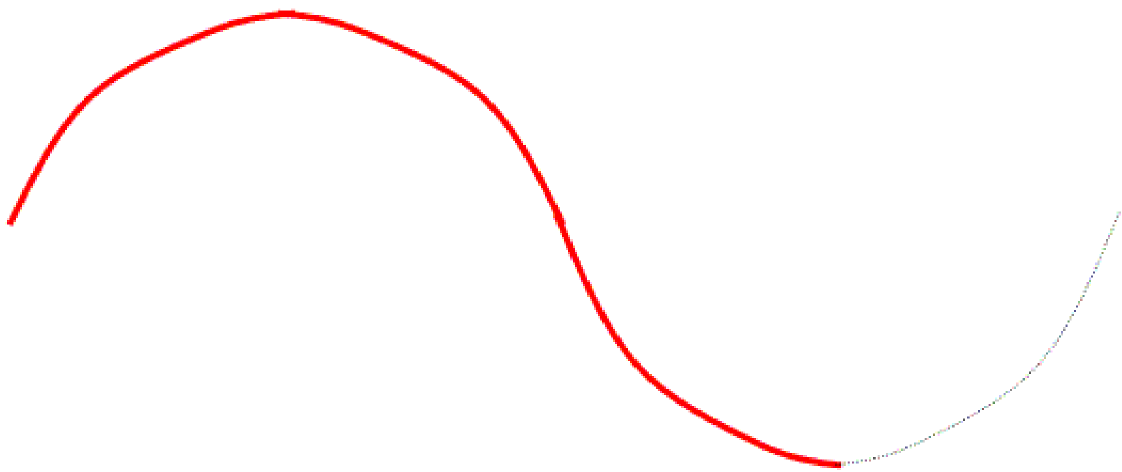

Illustration 4: Three quarters of a wavelength (3/4 λ)

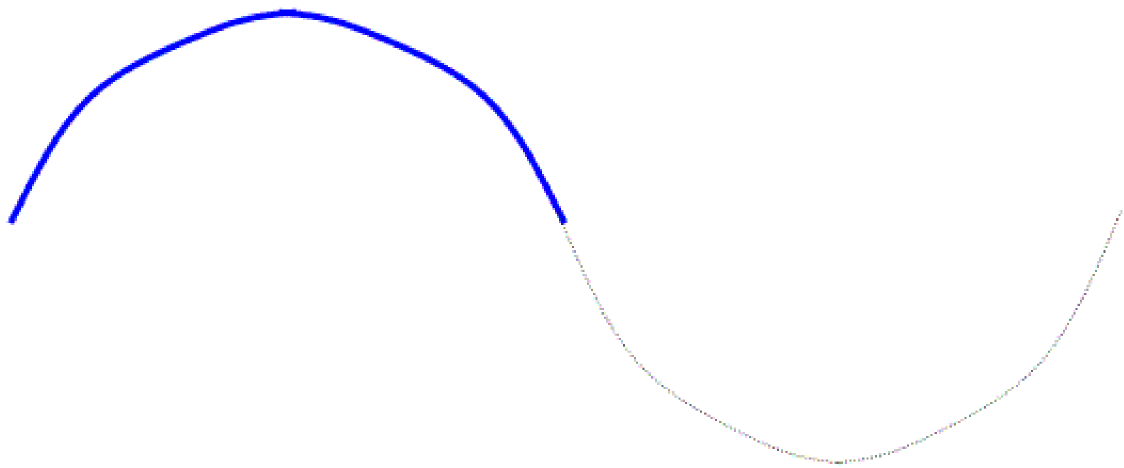

Illustration 5: One half of a wavelength (1/2 λ).

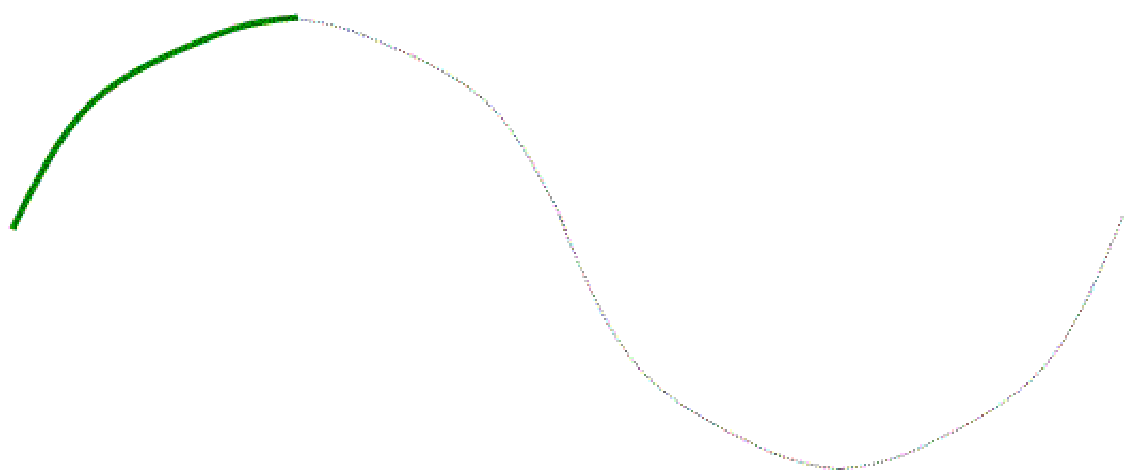

Illustration 6: One quarter of a wavelength (1/4 λ).

These wave fractions might appear upside down, flipped over, turned around, etc. in the diagrams you will see, but they will still represent the same portions of a wave.

When we are talking about the sounds that columns can make, what we are really concerned with is how much of the wave we can fit into the column.

- Different amounts of a wavelength in a column will result in a different frequency being heard.
- Because these are the frequencies of the waves that will naturally resonate in the columns, we call them the **resonant frequencies**.
    - In music, you might have heard these referred to as **harmonics**.

Although the actual length of the column remains the same, different notes are played.

- If you have ever played a wind instrument, you know that with the same keys pressed you can play different notes by controlling how you blow into the instrument… the difference between let's say a low C and a high C note.
- The lowest note you can play (which is also the smallest part of the wave that can fit inside the

column) is usually called the **fundamental**.
  - If you have no reason to believe otherwise, you should always assume that you are supposed to do your calculations for the **fundamental**.
- Fitting in more of the wave produces different notes, different **harmonics**.
  - **Harmonic** frequencies will always be multiples of the **fundamental.**

Closed Ended Columns

Remember that it is actually air that is doing the vibrating as a wave here.

- The air at the closed end of the column must be a node (not moving), since the air is not free to move there and must be able to be reflected back.
- There must also be an antinode where the opening is, since that is where there is maximum movement of the air.

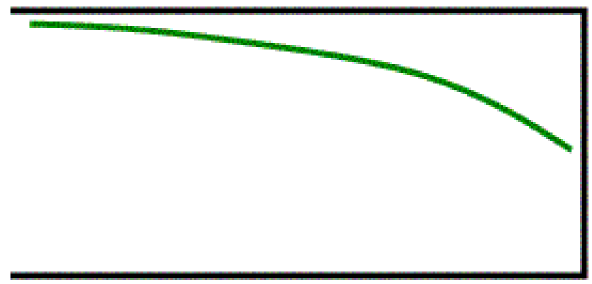

Illustration 7: Closed ended column, fundamental.

Fundamental (First Harmonic)

The simplest, smallest wave that I can possibly fit in a **closed end column** is shown in *Illustration 7*.

- Notice how even though it has been flipped left-to-right and it looks squished and stretched a bit to fit, this is still ¼ of a wavelength.
- Since this is the smallest stable piece of a wave I can fit in this column, this is the **Fundamental**, or **1st Harmonic**.

Since the length of the tube is the same as the length of the ¼ wavelength, I know that the length of this tube is ¼ of a wavelength… this leads to our first formula:

$$L=\frac{1}{4}\lambda$$

- "L" is the length of the tube in metres. On it's own this formula really doesn't help us much.
- Instead, we have to solve this formula for λ and then combine it with the formula v=fλ to get a more useful formula:

$$L=\frac{1}{4}\lambda \rightarrow \lambda=4L$$

$$v=f\lambda$$

$$v=f(4L)$$

$$f=\frac{v}{4L}$$

f = frequency of sound (Hz)
v = velocity of sound in air (m/s)
L = length of tube (m)

When the wave reaches the closed end it's going to be reflected as an inverted wave (going from air to whatever the column is made of is a pretty big change

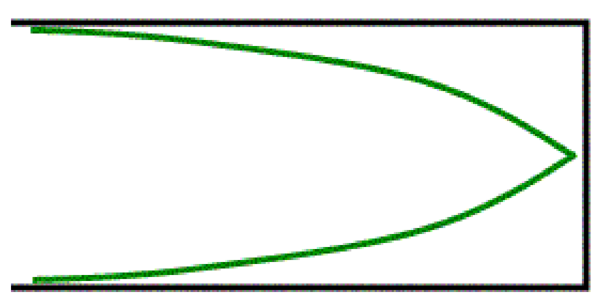

Illustration 8: Fundamental for a closed tube, showing reflection.

so this is what we would expect. It would look like *Illustration 8*.

- This does not change the length of the wave in our formula, since we are only seeing the reflection of the wave that already exists in the column.

Third Harmonic

What does the next harmonic look like? Well, for starters it's the **3rd Harmonic**.

- I know this name might seem a little confusing, but because of the actual notes produced and the way the waves fit in, musicians refer to the next step up in a closed end column instrument as the **3rd harmonic**… there is no such thing as a **2nd harmonic** for closed end columns.
- In fact, **all of the harmonics in closed end columns are going to be odd numbers**.

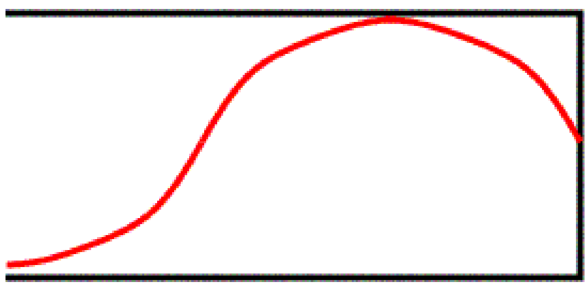

Illustration 9: Third harmonic for a closed end.

Remember that we have to have an antinode at the opening (where the air is moving) and a node at the closed end (where the air can't move). That means for the **3rd harmonic** we get something like *Illustration 9*.

- This is ¾ of a wavelength fit into the tube, so the length of the tube is…

$$L=\frac{3}{4}\lambda$$

- This is the **third harmonic**. The formula for the frequency of the note we will hear is…

$$f=\frac{3v}{4L}$$

Do you notice a pattern forming in the formulas? Hopefully, because for both open and closed end columns, we will only give you the formulas for the fundamentals. You need to remember how to get the rest.

- Notice how all the formulas for the closed end columns are something over **four**.
- The number on top is always an odd number that is the same as the harmonic.
- Let's see if the pattern continues.

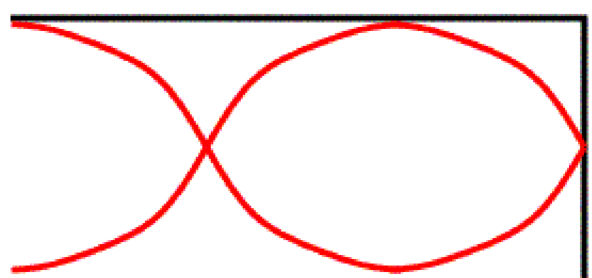

Illustration 10: Third harmonic with reflection.

Fifth Harmonic

One more to make sure you see the pattern. The **5th Harmonic** (*Illustration 11*).

- There is one full wavelength in there (4/4) plus an extra ¼ of a wavelength for a total of 5/4.

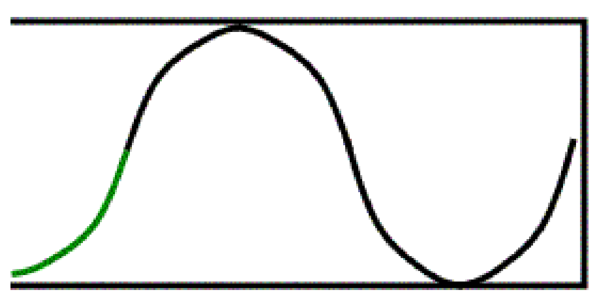

Illustration 11: Fifth harmonic of a closed column.

The length of the column is…

$$L=\frac{5}{4}\lambda$$

And the note produced by the 5th Harmonic is found using the formula…

$$f=\frac{5v}{4L}$$

Warning!
It is very easy to screw up the calculations using these formulas by being sloppy with your calculator. Make sure to put brackets around everything on the bottom of the formula so that order of operations is done correctly.

Open End columns

First Harmonic

I know you're probably thinking that there couldn't possibly be any more stuff to learn about this, but we still have to do **open end columns**. Thankfully, they're not that hard, and if you got the basics for **closed columns** it should go pretty fast for you.

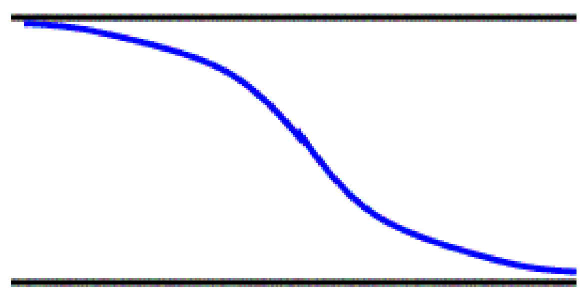

Illustration 12: Fundamental for an open end column.

- The **fundamental** (**first harmonic**) for an **open end column** needs to be an antinode at both ends, since the air can move at both ends.
    - That's why the smallest wave we can fit in is shown in *Illustration 12*.
    - This looks different than the ½ wavelength that I showed you in *Illustration 5*, but it is still half of a full wavelength.
- That means the length of the tube and frequency formula are…

$$L=\frac{1}{2}\lambda$$

$$f=\frac{v}{2L}$$

- The whole thing after it reflects at the other end looks like *Illustration 13*.

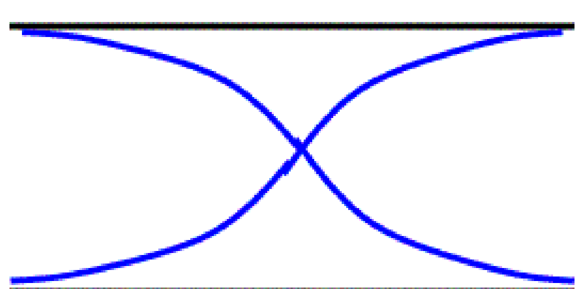

Illustration 13: Fundamental for an open column, showing reflection.

Second Harmonic

The next note we can play is the **2nd harmonic**.

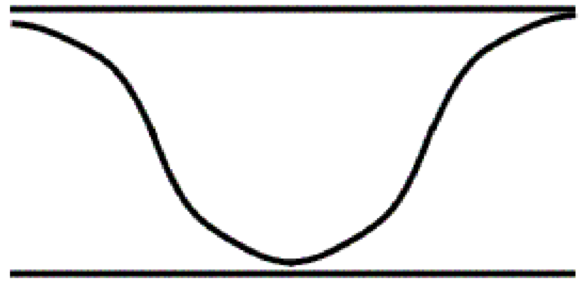

Illustration 14: Second Harmonic of an open end column.

- Yep, open end columns have a **2nd harmonic**… **open end columns can have any number harmonic they want, odd or even**.
- Again, it kind of looks weird, but trace it out and you'll see that there is exactly one wavelength here.
- The length and frequency formulas are…

$$L = \frac{2}{2}\lambda$$
$$f = \frac{2v}{2L}$$

I know that 2 over 2 equals 1, so it isn't really necessary, but I've left it in so you can see the pattern in these formulas.

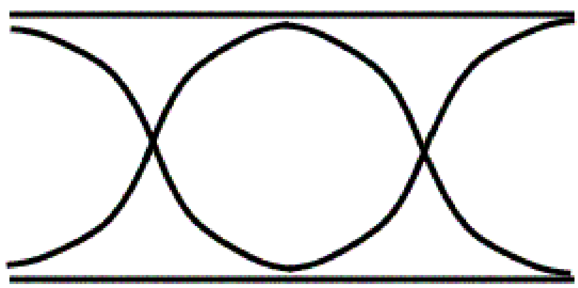

Illustration 15: Open end column second harmonic, showing reflection.

Third Harmonic

I bet you can guess what the formulas for the **third harmonic** looks like...

$$L = \frac{3}{2}\lambda$$
$$f = \frac{3v}{2L}$$

- The pattern is still continuing...
  - The number on top can be any even or odd number, the number of the harmonic.
  - The bottom number is always 2 for **open ended columns**.

Summary

Harmonic	Closed End Column	Open End Column

Fundamental (1st Harmonic)	$L=\frac{1}{4}\lambda \quad f=\frac{v}{4L}$	$L=\frac{1}{2}\lambda \quad f=\frac{v}{2L}$
2nd Harmonic		$L=\frac{2}{2}\lambda \quad f=\frac{2v}{2L}$
3rd Harmonic	$L=\frac{3}{4}\lambda \quad f=\frac{3v}{4L}$	$L=\frac{3}{2}\lambda \quad f=\frac{3v}{2L}$
4th Harmonic		$L=\frac{4}{2}\lambda \quad f=\frac{4v}{2L}$
5th Harmonic	$L=\frac{5}{4}\lambda \quad f=\frac{5v}{4L}$	$L=\frac{5}{2}\lambda \quad f=\frac{5v}{2L}$

* The table could continue, although in real life there are practical limits on how high a harmonic a person can actually play on an instrument.

Example 1: An open ended organ column is 3.6m long.

a) **Determine** the wavelength of the fundamental played by this column.
b) **Determine** the frequency of this note if the speed of sound is 346m/s. (Calculate it using the formulas you've just learned, although if you wanted you could use v = f λ)
c) **Determine** the note that could be played at the third harmonic on this column.
d) If we made the column longer, **explain** what would happen to the fundamental note… would it be higher or lower frequency?

a)
$$L=\frac{1}{2}\lambda$$
$$\lambda=2L$$
$$\lambda=2(3.6)$$
$$\lambda=7.2\text{m}$$

b)
$$f=\frac{v}{2L}$$
$$f=\frac{346}{2(3.6)}$$
$$f=48Hz$$

c) Notice that in the formula the third harmonic is three times bigger than the first harmonic. Since you have already calculated the first harmonic, you could just multiply that answer by 3 if you wanted to. Instead, we'll show the entire calculation here, just to make sure.

$$f=\frac{3v}{2L}$$
$$f=\frac{3(346)}{2(3.6)}$$
$$f=144Hz=1.4e2Hz$$

d) If we made the column longer, the wavelength would be bigger (just look at the formula in part (a) of this example), and since wavelength and frequency are inversely related, that means the frequency would be smaller.