

# Lesson 48: Superpositioning of Waves

Have you ever played around with pictures on your computer? Maybe you put a picture of your brother's face on a bird's body?

- I did this picture of my daughter... yeah, I have high hopes for her.
- I **superimposed** her face onto a picture of an astronaut.
- **Superimposed** just means to put something on top of something else, or blend two things together.
- **Superimposing** destroys some of the original information. Looking at this picture, you don't know what the rest of my daughter looked like in the original image, or what the face of the real astronaut looks like... that information is gone forever!



*Illustration 1: Katrien the astronaut!*

Waves can do a similar trick, but when they pass over each other they only have a **temporary** effect on each other.

- As soon as they are past each other, they look exactly the same as they originally did individually.
- That's why we give it a different name... **superpositioning**.

We will follow a very simple rule (even if it sounds confusing), called the “**Principle of Superposition**”  
*“The displacement of a medium caused by two or more waves is the algebraic sum of the amplitudes of the waves”*

What does this mean?

- “*Displacement of the medium*” just means how big the amplitude of the waves will be.
- “*Algebraic sum*” is just the simple addition of numbers, like  $2 + 3 = 5$  or  $4 + -3 = 1$ . This is how we will add the amplitudes.

The waves can either meet and create a bigger wave than they were individually (**constructive interference**), or they can create a smaller wave (**destructive interference**).

## Constructive Interference

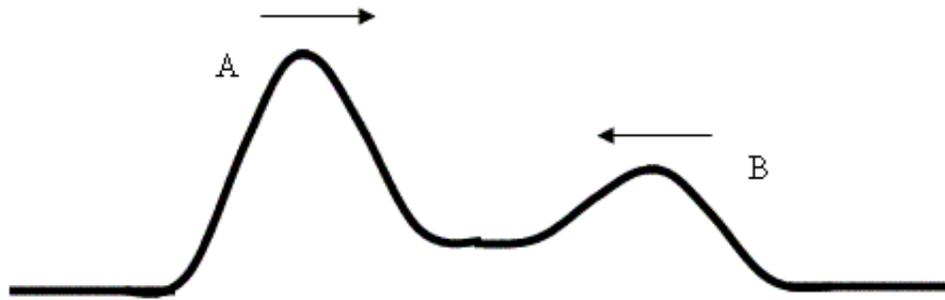
**Example 1:** The two wave pulses shown below are moving towards each other. **Sketch** what will happen when they interact.



*Illustration 2: Two waves approaching each other.*

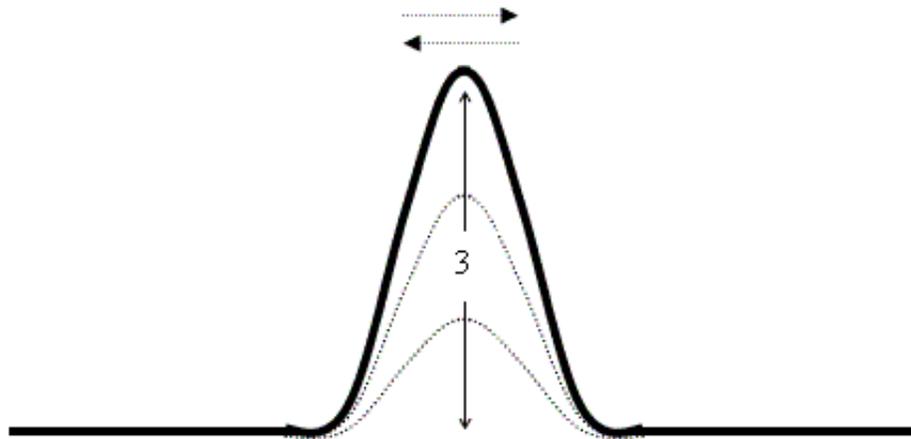
- Notice that wave **A** has an amplitude of 2, while wave **B** has an amplitude of 1.
- Both of the wave pulses are erect, so we say that they have positive values for amplitudes.

- As they come together in the middle, both of them are pulling particles of the medium upwards...



*Illustration 3: The waves start to collide in the middle.*

- When they are directly over each other, they are both shoving particles up together, so the two waves become one big wave with an amplitude of 3 for an instant of time.



*Illustration 4: The two waves are overlapping to form one wave.*

- They are still two separate waves, they just happen to be in the same spot at the same time.
- Afterwards, they will continue moving on and look exactly the way they looked before they hit each other.



*Illustration 5: The two waves have passed each other.*

This is an example of **constructive interference**.

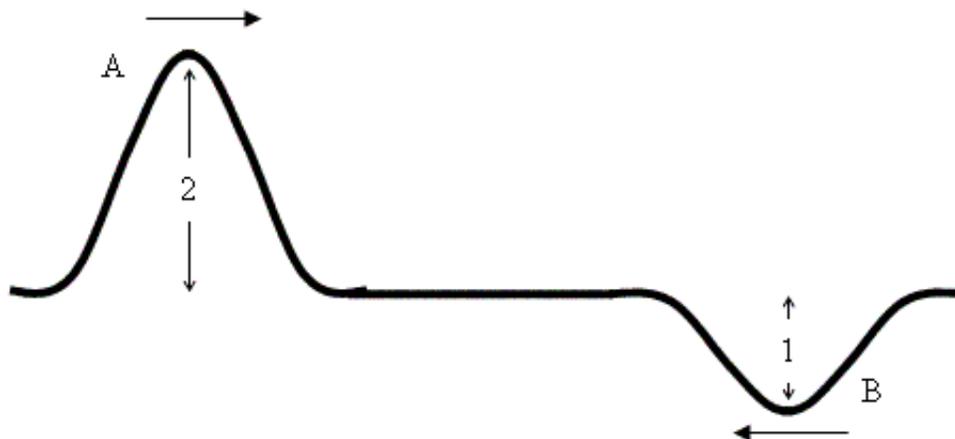
- The two waves interfered with each other, but did it in such a way that they **constructed** something bigger than either of them alone.

We usually think of things that are hitting each other as destroying themselves permanently.

- This is not the case for waves, since the waves themselves are not made of matter, they only travel through matter.
- Waves might temporarily “destroy” each other, but as soon as they pass each other they will go back to their original appearance.

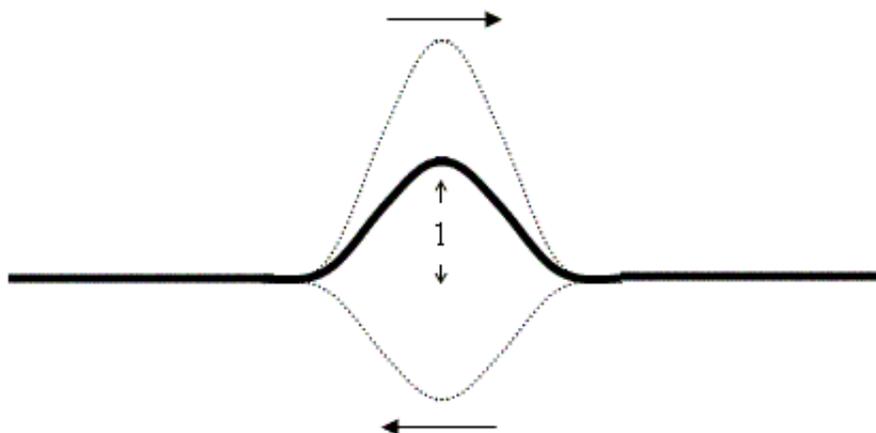
## Destructive Interference

**Example 2:** The following two wave pulses are going to collide. **Sketch** what will happen.



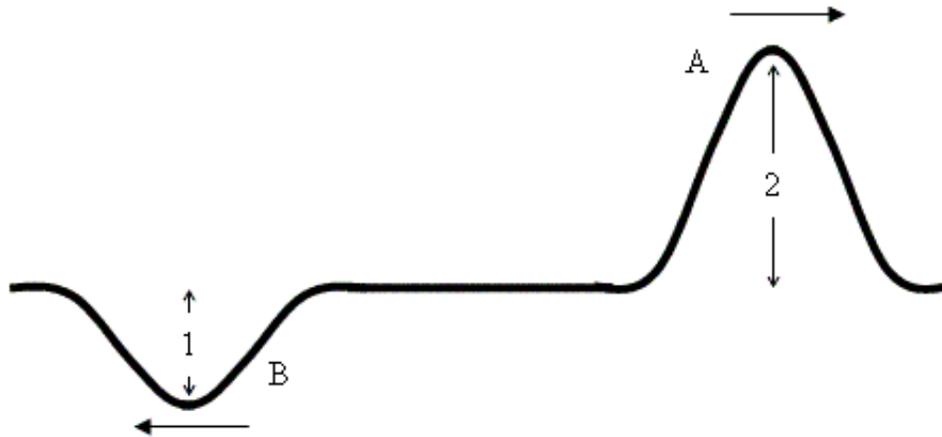
*Illustration 6: Two waves pulses approach each other.*

- Notice that **A** and **B** are still the same amplitude as in Example 1, but now **B** is *inverted*.
- As they pass over each other they would look like this...



*Illustration 7: The two pulses interact with each other.*

- For a moment the two wave pulses become one small wave pulse with an amplitude of  $(+2 + -1 = +1)$  positive one.
- As soon as they have passed each other...



*Illustration 8: The waves have passed each other.*

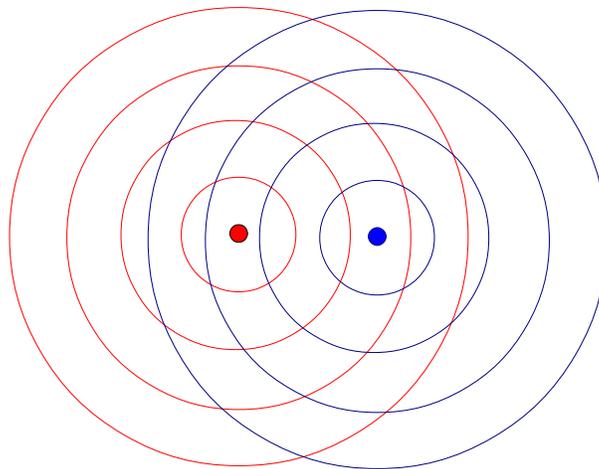
- Since the two waves together were smaller than they would be on their own, we call this **destructive interference**.

## Interference from Two Point Sources

What if we start off with two separate point sources making waves in-phase?

- In-phase just means that, even though they are separate, they are doing the same thing at the same time.
  - For example, as one of the sources is making a crest, the other source is also making a crest.
  - The two sources will also be creating waves that are identical in wavelength, frequency, velocity, and amplitude.

Look at how the crests and troughs of the waves produced by the two point sources interact in the following diagram.



*Drawing 1: Top view of the wave trains created by two point sources.*

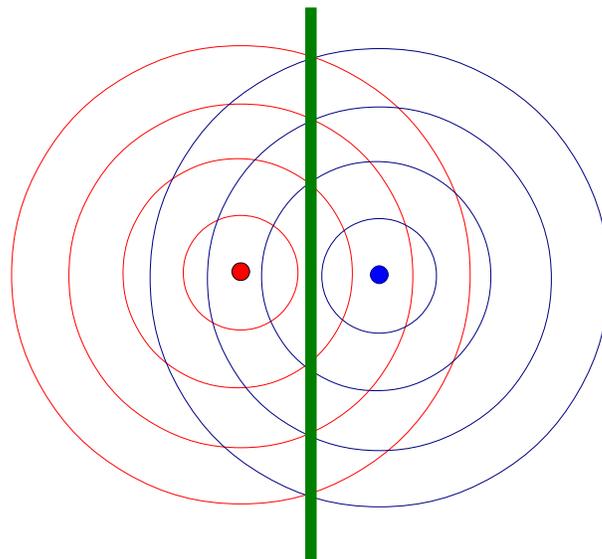
- You'll notice that the two point sources are exactly the same in every way, and that their waves cross over each other in a very regular pattern.

- What we must examine is the way that the crests and troughs pass each other to cause either constructive or destructive interference.

### Central Maximum

The easiest section to spot happens straight down the middle between the two point sources.

- If we draw a line down the middle between them, we find that all along the line **crests from one point** exactly overlap **crests from the other point**.
- The same can be said for the troughs.
- What we get here is a perfect case of **constructive interference**.
  - The crests will be really tall.
  - The troughs will be very deep.
- The crests and troughs created by each point source have to travel exactly the same distance to get to any location along this line, which is why they always arrive in-phase (crest-to-crest or trough-to-trough) with each other.
- Because of this perfect alignment of maximum crests and troughs, this line is called the **central maximum**.
  - You can also refer to it as **antinodes**.

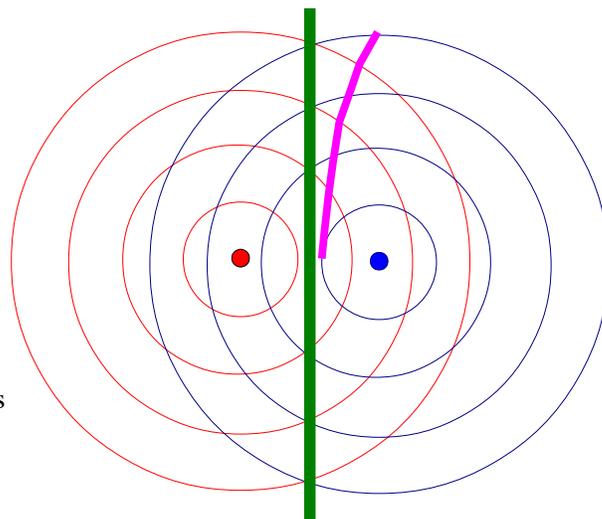


*Drawing 2: Central maximum.*

### Minimal Lines

If we go just a little bit (to either side) of the **central maximum**, we see a very different interaction between the waves from the two point sources.

- If we were to look, for example, just a little to the right of the **central maximum**, we can see the **crests from one point** meet the **troughs from the other**.
  - The **troughs from one** meet the **crests from the other** also.
- All along this line the crests meeting troughs will cause **destructive interference**. Since the amplitudes were the same, we can expect that the interference will be perfect and result in **no** wave visible along this line.
- Since there is no wave seen along this line, it is called a **minimum line** (or **nodal line**).



*Drawing 3: A minimum, or nodal, line.*

### Other Maxima and Minima Lines

If we kept on going outwards from the central maximum, we would find another line we could draw that again has **crests** meeting **crests** and **troughs** meeting **troughs**.

- This would create more waves that had really big crests and troughs.
- The difference is that they are no longer along the **central maximum**, so we would call them by names that are “orders.”
  - As the order number increases, you are further away from the **central maximum**.
- So, just off to the side of the **minimum line** described above, we would find the **first order maximum** (first out from the **central maximum**).
- As we go further outwards, we would find another line along which **crests** meeting **crests** and **troughs**

- meeting **troughs** again, and this would be the **second order maximum**.
- And so on, and so on...

In between the **maxima** we would also find a repeat of the crests meeting troughs.

- This would result in more **destructive interference** creating **minima** lines.
- This pattern would continue as we move out from the **central maximum**, alternating between **maxima** and **minima**.

The plural form of **maximum** is **maxima**, and the plural of **minimum** is **minima**.

If the waves being produced by the two point sources are not exactly the same, then these **maxima** and **minima** can start to have very complicated patterns.

- By analyzing these complex patterns, often using a computer, scientists can make predictions about the original waves that were involved.
- Seismologists use these sort of methods when analyzing the waves created in the earth during earthquakes.
  - The waves reflect, refract, and diffract because of the different layers of the Earth and then interact with each other.
  - The analysis can tell us things about the earthquake and the structure of the Earth itself.