Lesson 47: Wave Velocity and Boundaries

Wave Velocity

The speed of a wave does not depend on the **amplitude** or **wavelength** of the wave.
- Instead, the speed of the wave is determined by the properties of the medium it is traveling in.
- Some examples of how the properties of the medium can affect the speed of the wave are:
    1. Speed of water waves depend on the depth of the water.
    2. Speed of waves in a rope depend on the force exerted on the rope and the weight of rope used.
    3. Speed of sound in air depends on the temperature of the air.
- A wave with a bigger **amplitude** does transfer more energy, but it will still travel at the same speed as a smaller **amplitude** wave in that same medium.

Basic Wave Velocity

If we look at the creation of a single wave pulse, we can determine the velocity of the wave.
- Let's say you are creating a transverse wave pulse in a skipping rope by flicking the end of it with your hand.
- The instant you start to flick your hand upwards, the pulse starts to travel away from you through the rope.
- You will continue to raise your hand until it gets to the highest point (this will be the **amplitude** of the wave), and then your hand starts to come back down.
- By the time your hand gets back to where it started (the equilibrium point), the wave pulse has traveled a certain displacement that is actually the length of the wave pulse.
- Since it traveled this certain length in a certain time, we can determine the velocity of the pulse.

Warning!
Notice that I said **length**, not *wave***length**. This is because we are only looking at a pulse for now, not a wave train.
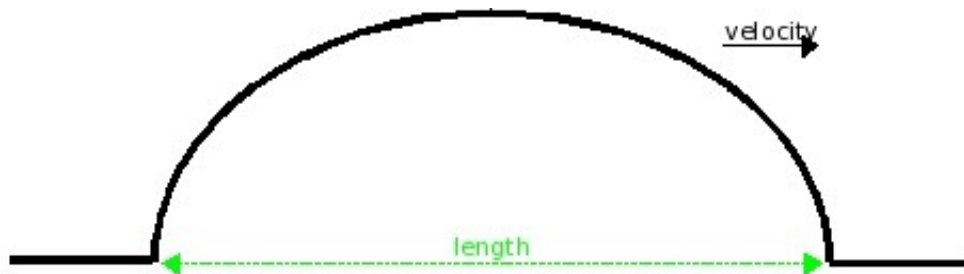

Illustration 1: A pulse, created in a skipping rope, traveling to the right.

$$v = \frac{d}{t}$$ since the displacement traveled is the length...

$$v = \frac{\ell}{t}$$

v = velocity (m/s)
ℓ = length (m)
t = time (s)

Example 1: It takes you 0.54s to flick your wrist up and down while holding a garden hose. If the pulse you create has a length of 0.38m, **determine** the velocity of the pulse.

$$v = \frac{\ell}{t}$$

$$v = \frac{0.38}{0.54}$$

$$v = 0.703703 = 0.70 \, m/s$$

Universal Wave Equation

Now let's assume that instead of making just a single wave pulse, you continue to flick your hand up and down to create a wave train of crests and troughs.

- Your hand will be flicking up and down with a certain **period** (time to go up and down once).
    - We could take the inverse of the period of your hand to find the **frequency** of your hand's motion.

$$T = \frac{1}{f}$$

- - This frequency is also the frequency of the wave you are producing!
    - Remember that frequency is measured in Hertz (Hz), which in this case means waves per second.
- We could measure how far the wave has traveled in the entire time it took you to raise and lower your hand one time, and we would find that this is actually the **wavelength** of the wave.
    - Remember that wavelength is just measured in metres.
- Take frequency and wavelength together and you can do a neat little trick.
    - If you multiply them, you are really multiplying cycles/second (frequency) by metres (wavelength), which gives you metres per second (velocity).

$$v = f \lambda$$

<div align="right">

v = velocity (m/s)
f = frequency (Hz)
λ = wavelength (m)

</div>

- This formula is so important, for any kind of wave, that it has been given the name **universal wave equation**.

Example 2: A wave in a hot tub is measured to have a frequency of 6.0Hz. If its wavelength is 24cm, **determine** how fast it is moving.

$$v = f \lambda$$

$$v = 6.0(0.24)$$

$$v = 1.4 \, m/s$$

Example 3: The speed of light is always 3.00e8 m/s. **Determine** the frequency of red light, which has a wavelength of 700nm.

$$v = f \lambda$$

$$f = \frac{v}{\lambda}$$

$$f = \frac{3.00e8}{700e\text{-}9}$$

$$f = 4.29e14 \; Hz$$

Waves at Boundaries

Quite often a wave will move from one medium to another, like sound traveling through the air and then into water.

- This may cause the wave to be somewhat distorted (*changed randomly*). We will assume no distortion happens in our examples.
- Moving to a different medium will definitely change the velocity of the wave
- Since the frequency of the source that created the wave remains unchanged, changing the velocity of the wave must result in the wavelength changing.

$$v = f \lambda$$

v	varies
f	constant
λ	varies

To fully explain what happens to the wave, we need to give names to the wave as it goes through changes in media.

- The original wave that was in the first medium = **incident wave**.
- The wave that continues into the new medium = **transmitted wave**.
- Any wave that bounces back = **reflected wave**.

Different things will happen when the incident wave hits the boundary between the two media, depending on the densities of the media compared to each other.

- The two ideas we will examine are when the difference in the density of the two media is large, and when the density difference is small. Big density changes can be divided into two other situations.

Difference in Density is BIG

If there is a big difference between the densities of the media (like solid to liquid, or gas to liquid), then almost all the wave is **reflected**. The following two situations can happen.

### Situation 1: Low density to higher density...

1. The **incident** wave is moving towards the boundary.
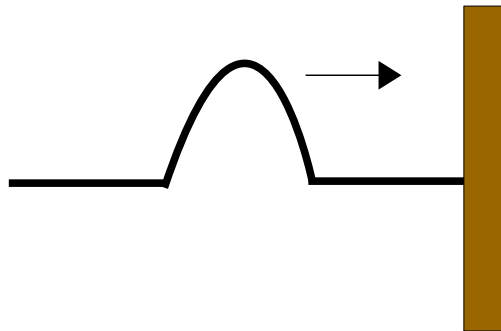
Note:
We are using a **spring** attached to a door in this example and treating it as a big density change. The spring acts nearly like a fluid (it will "fill" a container I put it into), while the door is solid.

2. When it hits the boundary, almost all of the wave will be **reflected** back the way it came in the spring! Almost none of the wave will be **transmitted** into the door.
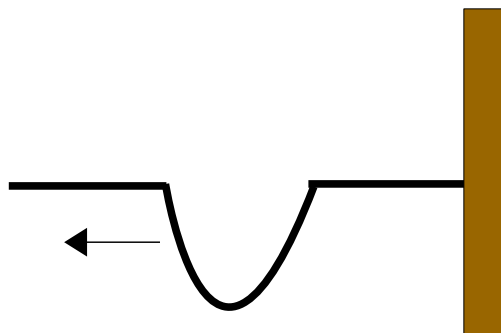
- Notice that the **reflected** wave is upside down (we call this ***inverted***), but any wave traveling into the door as the **transmitted** wave will still be upright (we call this ***erect***). We probably just see the door shake a bit.
- The **reflected** wave will be traveling at the same speed, and the **transmitted** wave will slow down.

Situation 2: High density to lower density...

1. The **incident** wave is moving towards the boundary.

Note:
Now the **spring** is attached to nothing, so the transmission is into the air. This is still a big density change, but now we are going from liquid (the spring) to gas.

2. At the boundary almost all of the wave is still **reflected** back in the spring. Only a tiny bit will be **transmitted** into the air.

- Notice that the **reflected** wave is upright (*erect*), and so will any wave traveling into the air as the **transmitted** wave. We probably just see the door shake a bit.
- The **reflected** wave will be traveling at the same speed, and the **transmitted** wave will speed up

Difference in Density is SMALL

If the change in density is small, things are a lot easier to remember.

- Quite simply, almost all of the incident wave will be **transmitted** and stay erect.
- In this example, we use two ropes that have almost the same density, but not quite.

- The **incident** wave will almost entirely **transmit**, since there is very little difference for it to even notice.

- That's it! The **transmitted** wave is (almost) the same as the original **incident** wave.
- Only a very little bit of the wave will be **reflected** (I didn't even bother drawing it in).

Homework
409 #1-3
410 #3, 4, 5, 7