Lesson 43: Simple Harmonic Motion

As far back as Lesson 31 we started talking about ideas like period and frequency, and more recently in Lesson 39 Hooke's Law.

- We can bring these ideas together now to look more carefully at the idea of **Simple Harmonic Motion** (**SHM**).
- **Simple harmonic motion** is any situation where an object...
    1. repeats its motion over and over again in a fixed pattern that has a certain period.
    2. there is a restoring force causing it to move back to its equilibrium point (where it was at rest before any force was applied to it). The restoring force is directly related to the amount of displacement from equilibrium (that's Hooke's Law).
- **SHM** happens to objects that are **simple harmonic *oscillators***.

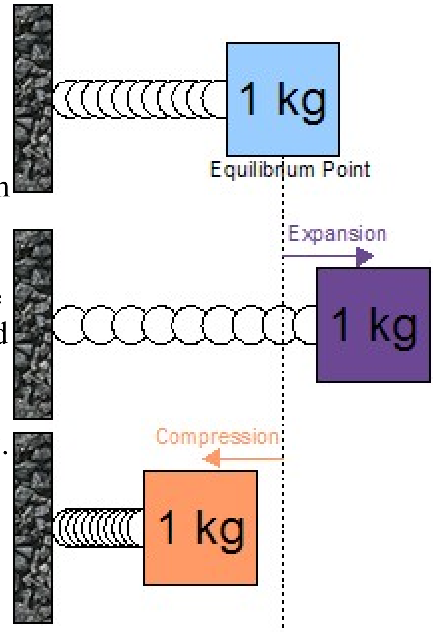

Illustration 1: A mass starts at its equilibrium point, is stretched outwards, and then bounces back and forth, including compressing past its equilibrium point.

If we look at situations where a mass is attached to an elastic object (like a spring), we can use our knowledge of the forces acting on it to start making predictions of how it will move.

- If the mass is stretched away from equilibrium, the restoring force will pull it back to equilibrium. The amount of restoring force would be calculated using $F_s = -kx$.
- Besides the restoring force (calculated using Hooke's Law) you may have to take other forces into account.
    - One example of this sort of situation would be a spring hanging vertically. Then you would need to take the force due to gravity into account.

Example 1: A spring is attached to the ceiling dangling downwards to make a basic scale for measuring masses. The spring constant is 38.9 N/m. At first it is just hanging there on its own at its equilibrium point. If a 1.3 kg mass is hung from the spring, **determine** how far it will stretch.

In this position, the amount of gravitational force pulling it down will be balanced by the restoring force pulling it up and the mass will just hang at its position with a net force of zero acting on it.

$$F_{NET} = F_s + F_g$$
$$0 = F_s + F_g$$
$$F_s = F_g$$
$$kx = mg$$
$$x = \frac{mg}{k} = \frac{1.3(9.81)}{38.9}$$
$$x = 0.32784062 = 0.33m$$

Skip the negative sign in the elastic restoring force and gravity, since in elastic situations the sign is based on expansion or compression, while in gravity its based on up and down directions.

Because it will just hang at this position on its own forever, this is its **new** equilibrium point ***with the mass attached***. We still need to calculate the restoring force based on the **original** equilibrium point.

Example 2: A person standing next to the scale with the mass attached tugs down on the mass and then lets go. **Sketch** a diagram showing the forces acting on the mass as it bounces up and down (SHM).

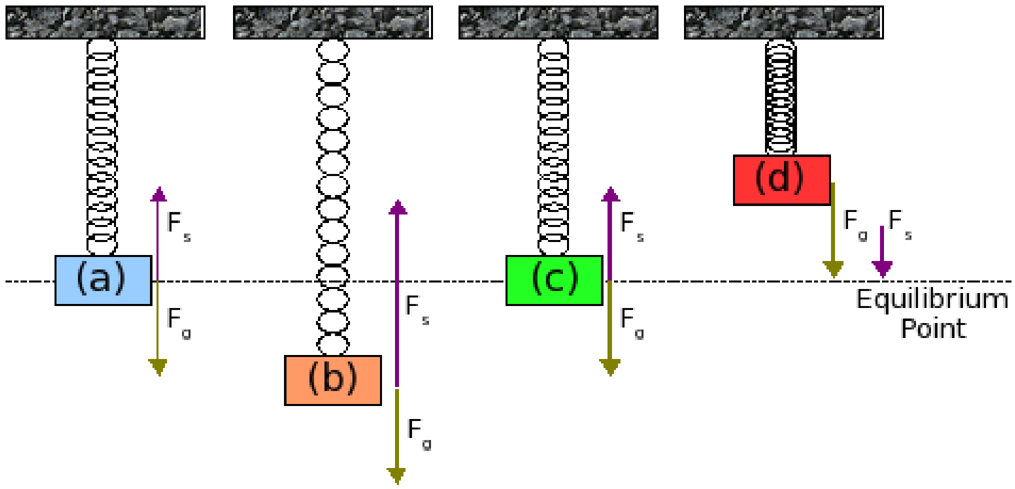

Illustration 2: Mass bouncing on spring.

(a) Be careful! This is the **new** equilibrium position that we were just talking about. The person hasn't pulled it yet, so $\mathbf{F_s}$ and $\mathbf{F_g}$ are equal and $\mathbf{F_{NET}}$ is zero. The mass will just hang there.

(b) The person has pulled the mass down and let go. Since it has been pulled *further* away from the **original** equilibrium point, $\mathbf{F_s}$ is bigger. $\mathbf{F_g}$ always stays the same. $\mathbf{F_{NET}}$ is upwards, pulling the mass upwards so it accelerates up.

(c) At this point $\mathbf{F_s}$ and $\mathbf{F_g}$ are equal again so $\mathbf{F_{NET}}$ is zero. This does not mean that the mass stops; instead it will continue moving the way it is already, which means it continues to move upwards.

(d) As long as the mass travels up higher than the original equilibrium point, the spring can be considered to be compressed and $\mathbf{F_s}$ will be pushing the mass down. $\mathbf{F_g}$ is also down, meaning that $\mathbf{F_{NET}}$ will be acting downwards, which will slow down and eventually stop the mass. Then the mass will begin to move downwards faster and faster.

Pendulums are also an example of SHM, although they are not perfect.

- When a pendulum is just sitting on its own at its equilibrium point, it has no reason to do anything at all.
- If it is pulled off to the side, there will be a restoring force trying to get it back to the equilibrium point.
  - This restoring force is actually a component of the force due to gravity.
- As the pendulum is pulled off to the side at a bigger and bigger angle, the restoring force does increase, but it does not do it in a steady, linear way.
  - Since the restoring force is ***not*** directly related to the displacement, we can't say it is SHM.
  - What we can do is keep the angle fairly small (less than about

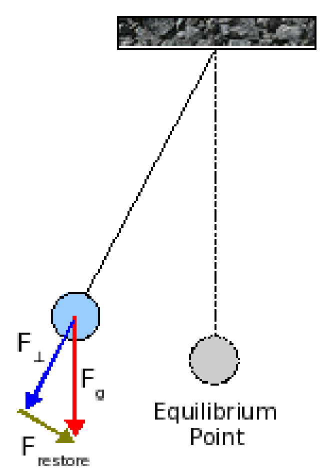

Illustration 3: Pendulum pulled off to the side.

15°), for which the force does increase almost linearly, so we can say it is SHM for angles up to 15°.

Example 3: A 2.0kg mass is attached to the end of a pendulum. If the pendulum is held out at a 10° angle from equilibrium, **determine** the restoring force.

If you look at Illustration 3, you'll see that if the pendulum is pulled off to the side at a 10° angle, the angle will also be 10° in the diagram for the components as shown in Illustration 4.

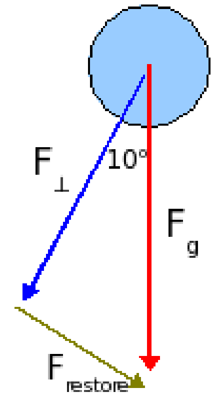

Illustration 4: Components of force due to gravity.

$$\sin\theta=\frac{opp}{hyp}$$

$$\sin\theta=\frac{F_{restore}}{F_g}$$

$$F_{restore}=\sin\theta F_g$$

$$F_{restore}=\sin 10^o(2.0)(9.81)$$

$$F_{restore}=3.40697725=3.4\,N$$