Lesson 40: Conservation of Energy

Total Mechanical Energy

We sometimes call the total energy of an object (potential and kinetic) the **total mechanical energy** of an object.

- "Mechanical" energy doesn't mean that it always has to involve machines.
  - An apple falling off a cliff has **gravitational potential** and **kinetic energy**, so it therefore has **mechanical energy**.

$$E_m = E_k + E_p$$

Example 1: A 14 300 kg airplane is flying at an altitude of 497 m at a speed of 214 km/h. **Determine** the airplane's total mechanical energy.

Remember to convert the speed into metres per second.

$$E_m = E_k + E_p$$
$$E_m = \frac{1}{2}mv^2 + mgh$$
$$E_m = \frac{1}{2}(14300)(59.\bar{4})^2 + 14300(9.81)(497)$$
$$E_m = 94\,986\,191 = 9.50e7\,J$$

Notice that the calculation and the answer have nothing to do with direction, since energy is ***scalar***.

We can treat this flying airplane as a **system**.

- A system is any collection of objects in a particular place that we have defined. We usually do this so we can describe a problem as having particular limits (e.g. "the ball is rolling down the slope.")
- Systems come in three main types, depending on what is conserved (kept constant) throughout the situation.

System	Matter	Energy
Isolated	**Conserved**	**Conserved**
Closed	**Conserved**	**Not Conserved**
Open	**Not Conserved**	**Not Conserved**

In many questions we assume that we are dealing with an **isolated system**.

- An isolated system simply means a situation where *nothing*, not even energy, can enter or leave. Whatever you started with you finish with.
  - Since isolated systems can't lose or gain energy, we have to be careful that **non-conservative forces** (such as friction) do ***not*** happen.
  - Also, if **work is done on or by the system** it will change the energy of the system. In that case it can not be an isolated system

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- As long as we deal with isolated systems we know that the total mechanical energy **before** and **after** will be equal.

$$E_m = E_m'$$
$$E_k + E_p = E_k' + E_p'$$

- So if the one type of energy decreases, the other type of energy will increase by a similar amount.
  - Energy is not being created or changed, it is only changing forms or transferring from one object to another. This is known as the **Law of Conservation of Energy**.

Example 2: A person is sitting on a toboggan at the top of a 23.7m tall hill. If the person and toboggan have a total mass of 37.3 kg , **determine** how fast they will be going when they reach the bottom of the hill. Assume there is no friction.

- If there was friction, then it would not be an isolated system. The frictional force would result in some heat being given off, which would be energy leaving the system.
- At the top of the hill the person isn't moving, so E_k will be zero. At the bottom of the hill the E_p will be zero because the person is zero metres above the reference point.

$$E_k + E_p = E_k' + E_p'$$
$$\tfrac{1}{2} mv^2 + mgh = \tfrac{1}{2} mv'^2 + mgh'$$
$$0 + (37.3)(9.81)(23.7) = \tfrac{1}{2}(37.3) v'^2 + 0$$
$$8.67e3 = 18.7 v'^2$$
$$v' = 21.5637195 = 21.6 \text{ m/s}$$

Notice how in this example all of the potential energy the object had at the **top** of the hill has been turned completely into kinetic energy at the **bottom**.

- It's also possible to analyze how the potential energy steadily changes into kinetic energy during a fall…

Example 3: Wille E. Coyote is trying to drop a 10 kg boulder off a 10 m high cliff to hit the Roadrunner eating a bowl of birdseed on the road below. At this particular location, gravity is 10 N/kg. He wants to know the speed of the boulder at various points above the road...

a) 10 m
b) 8.0 m
c) 3.0 m
d) 0 m

Don't worry too much about sig digs here.

a) Well, this one ain't so tough! Since it's sitting at the top of the cliff, its velocity is 0 m/s. It might be handy at this point to calculate how much E_p the boulder has at this time, since this is also the total mechanical energy it starts out with!

$$\left.\begin{array}{l} E_p = mgh = 10(10)(10) = \mathbf{1000\ J} \\ E_k = 0 \end{array}\right\} \textbf{Total Mechanical Energy = 1000 J}$$

b) First, ask yourself how much E_p the boulder still has at 8.0 m above the ground.

E_p = mgh = 10 (10) (8.0) = **800 J**

That means that **1000 J** - **800 J** = **200 J** is missing, right? Well, not missing, just changed. According to the conservation of mechanical energy, that energy must now be **kinetic**!

$$\left.\begin{array}{l} E_p = \mathbf{800\ J} \\ E_k = \mathbf{200\ J} \end{array}\right\} \textbf{Total Mechanical Energy = 1000 J}$$

$$E_k = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{2 E_k}{m}}$$

$$v = \sqrt{\frac{2(200)}{10}}$$

$$v = 6.32456 = 6.3 m/s$$

So it's falling at 6.3 m/s at this time.

c) Again, calculate how much E_p you have at this new height of 3.0 m, then figure out how much is kinetic so they both add up to 1000 J total...

$$\left.\begin{array}{l} E_p = mgh = 10\,(10)\,(3.0) = \mathbf{300\ J} \\ E_k = \mathbf{700\ J} \end{array}\right\} \textbf{Total Mechanical Energy = 1000 J}$$

$$E_k = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{2 E_k}{m}}$$

$$v = \sqrt{\frac{2(700)}{10}}$$

$$v = 11.8322 = 12 m/s$$

So it's falling at 12 m/s at this time.

d) By the time the boulder has reached the ground, all of its potential energy is gone (it's zero metres above the ground!). We all know that when it actually hits the ground it will come to rest, but we are concerned with how fast it's going when it is right at ground level but hasn't actually touched the ground yet. We can assume that all of the potential energy the boulder had at the top is now kinetic energy at the bottom…

$$\left.\begin{array}{l} E_p = \mathbf{0\ J} \\ E_k = \mathbf{1000\ J} \end{array}\right\} \textbf{Total Mechanical Energy = 1000 J}$$

$$E_k = \frac{1}{2} m v^2$$
$$v = \sqrt{\frac{2 E_k}{m}}$$
$$v = \sqrt{\frac{2(1000)}{10}}$$
$$v = 14.1421 = 14 m/s$$

So it's falling at 14 m/s at the moment it hits the ground.

You could be finding the same answers based on kinematics formulas from earlier lessons.

- In fact, you'll find that conservation of energy gives you new ways to do many problems that you did with kinematics formulas.

More Examples

Example 4: A toy car with a mass of 112g is pushed by a student along a track so that it is moving at 3.00 m/s. It hits a spring (k = 925 N/m) at the end of the track, causing it to compress.

a) **Determine** how far did the spring compress to bring the car to a stop.
b) If the spring only compressed 2.00 cm in bringing the car to a stop, **explain** what happened.

a) We know that conservation of mechanical energy means that the kinetic energy of the car moving will be turned into elastic potential energy stored in the spring. So…

$$E_k + E_p = E_k' + E_p'$$
$$E_k = E_p'$$
$$\frac{1}{2} m v^2 = \frac{1}{2} k x^2$$
$$x = \sqrt{\frac{m v^2}{k}} = \sqrt{\frac{0.112(3.00^2)}{925}} = 0.033011 = 0.0330\, m$$

b) If the spring didn't get compressed as much as predicted, it must mean that some of the energy wasn't transferred perfectly… there must have been some waste energy released, maybe as the car rolled along the track or when the spring was compressed. *This is not an isolated system.*

To calculate how much energy was released, first figure out the total mechanical energy the car started with (that would be its kinetic energy)...

$$E_k = \frac{1}{2} m v^2$$
$$E_k = \frac{1}{2} 0.112(3.00)^2$$
$$E_k = 0.504\, J$$

...compared to how much energy it ended with (the elastic energy the spring did absorb)…

$$E_p = \frac{1}{2} kx^2$$

$$E_p = \frac{1}{2} 925(0.0200)^2$$

$$E_p = 0.185\,J$$

The amount of energy lost is...

$$\Delta E = E_f - E_i$$

$$\Delta E = 0.185 - 0.504$$

$$\Delta E = -0.319\,J$$

So there was 0.319J of energy "lost" to the surroundings.

Example 5: A pendulum is pulled aside and then released as shown in *Illustration 1*. **Determine** its speed at the bottom of the swing.

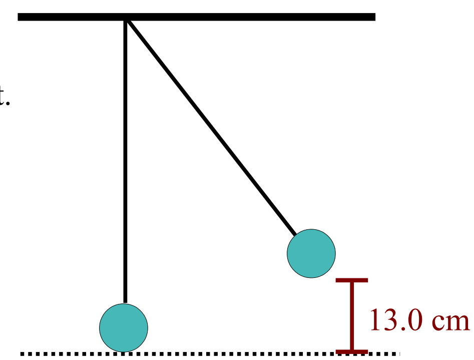

Illustration 1: Pendulum pulled to the side.

Even though this may look like a difficult problem, it really isn't. All we need to do is keep in mind that energy is conserved. The pendulum bob is 13.0cm above a reference point (it doesn't matter if it swings to get there), so it has potential energy. When it gets to the bottom of its swing, all that energy will have become kinetic energy.

$E_k + E_p = E_k' + E_p'$

$E_p = E_k'$

Cancel the masses.

$mgh = \frac{1}{2} m(v')^2$

$gh = \frac{1}{2} (v')^2$

$$v' = \sqrt{2gh} = \sqrt{2(9.81)(0.130)} = 1.59706 = 1.60\,m/s$$

Example 6: Same pendulum as in **Example 5**, but now we want to **determine** how fast it is moving when it is still 2.00 cm above the bottom of the swing.

So instead of letting it swing all the way to the bottom (where it has all kinetic and no potential energy), we look at the speed as it swings down but is still 2.00 cm above our reference line.

$E_k + E_p = E_k' + E_p'$

Notice we will still have some potential at the end.

$E_p = E_k' + E_p'$

$mgh = \frac{1}{2} m(v')^2 + mgh'$

$gh = \frac{1}{2} (v')^2 + gh'$

$$v' = \sqrt{2g(h-h')} = \sqrt{2(9.81)(0.130-0.0200)} = 1.46908 = 1.47\,m/s$$

Example 7: Last pendulum question, I promise! Instead of having the regular information about the height of a pendulum, you have been given the information as shown in *Illustration 2*. The string is 0.750 m long and held 40° away from the vertical. If it is released from this point, **determine** the speed of the pendulum at the bottom.

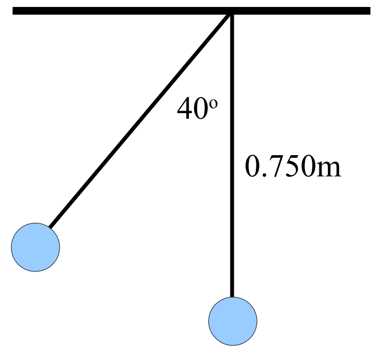

Illustration 2: Another (sigh) pendulum.

The tough part on this one is figuring out how high the pendulum is above its reference line. We're going to have to start with some trig, then solve the problem using energy later.

Start by drawing a triangle as shown in *Illustration 3*. Notice how we are using the pendulum swung out at 40o as the hypotenuse, and we know that it is 0.750 m long. The string of the pendulum will always be 0.750 m long no matter where we hold it. Solve for the adjacent side that I've labeled **y**.

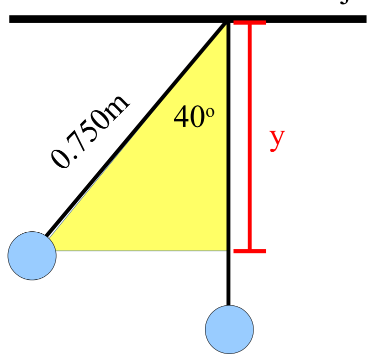

Illustration 3: Figuring out the adjacent side, y.

$$\cos\theta = \frac{adj}{hyp}$$
$$adj = \cos\theta(hyp)$$
$$adj = \cos 40^o(0.750)$$
$$adj = 0.574533\,m = y$$

Now remember that even when the pendulum is at its lowest point the string is still a total of 0.750 m long, so...

0.750 = **0.574533** + **h**
h = **0.750** – **0.574533**
h = **0.175467 m**

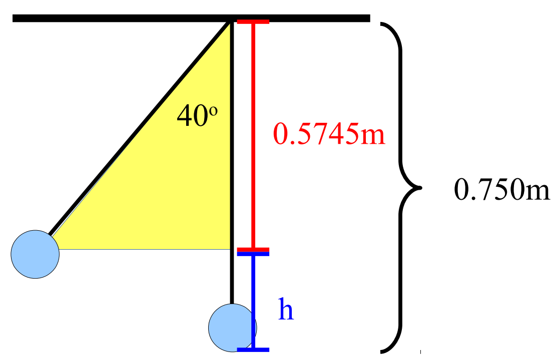

Illustration 4: Finding the height.

Now solve the rest of it just like **Example 5**.

$$v' = \sqrt{2gh} = \sqrt{2(9.81)(0.175467)} = 1.85544 = 1.9\,m/s$$

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Example 8: This question is an extension of the "Atwood's Pulley" question we did in Lesson 24: Newton's Second Law. The 7.50 kg box starts on the floor, and the 12.00 kg box is 3.00 m above the floor. The boxes are released; **determine** the speed of the 7.50 kg box as the 12.0 kg box hits the floor.

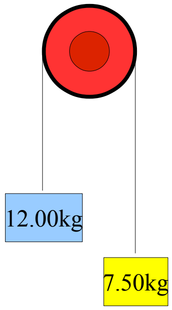

Illustration 5: Atwood's Pulley again!

Method 1: Kinematics

If you look back to the question in Lesson 24, you'll see that we figured out the acceleration (using dynamics) as 2.26 m/s^2. We can use that to figure out the velocity. (*Note, if we figure out one box's velocity, the other will be the same; it's just that one goes up while the other comes down.*)

$$v_f^2 = v_i^2 + 2ad$$
$$v_f = \sqrt{2ad} = \sqrt{2(2.26)(3.00)}$$
$$v_f = 3.69\, m/s$$

Method 2: Conservation of Energy

The total mechanical energy before anything starts to move must be equal to the total mechanical energy at the end. Before they start to move, only one of the masses has any energy; the 12.00 kg box is 3.0m in the air. Its potential energy is the total mechanical energy...

Before

$E_T = E_p = mgh = (12.00)(9.81)(3.0) = 353.16$ J

At the end both boxes will have kinetic energy, but we don't know how much (since we don't know the velocity). We do know that when the 12.00 kg box hits the floor, the 7.50 kg box will be 3.0m in the air. We can easily calculate how much potential energy the 7.50 kg mass will have at that time.

After

$E_p = mgh = (7.50)(9.81)(3.0) = 220.725$ J

Which means that we are unable to account for 353.16 – 220.725 = 132.435 J of energy at the end. This must be the kinetic energy of the two two masses as they are moving at the end. We need to divide that energy between the two masses, but since they have unequal masses the energy will be divided between them in a ratio based on their masses.

12.00 (x) + 7.50 (x) = 132.435 J
19.50 x = 132.435 J
x = 6.792 J

So the 7.50kg box has 7.50 (6.792 J) = 50.94 J of kinetic energy. Figuring out its speed is easy now…

$$E_k = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{2 E_k}{m}}$$

$$v = \sqrt{\frac{2(50.94)}{7.50}} = 3.69\ m/s$$

This is the velocity of both boxes, just in opposite directions.

Homework

p309 #2
p313 #2
p315 #2