Lesson 36: Satellites

In our modern world the world "**satellite**" almost always means a human made object launched into orbit around the Earth for TV or phone communications.

- This definition of satellites has really only been around since the 1950's when the Russians launched the first artificial satellite, Sputnik 1, into orbit around the Earth.
- The word **satellite** actually applies to any object that is in orbit around a (typically) larger object.
  - By this definition, the moon is a satellite of the Earth, and the Earth is a satellite of the sun.
- The good news is that basic ideas about satellites can be applied to just about any satellite (with some modifications).
  - The basics that you will learn here are the same general ideas that NASA and other space organizations use when launching artificial satellites into orbit, or studying natural satellites.

Illustration 1: Sputnik 1

Newton's Cannon

Yep, Newton again! We need to talk about Newton for about the third time in this course, but this should give you an idea of just how much stuff he studied and what a large impact he had on physics.

Newton came up with what seems like a very strange idea. He asked the question "*What would happen if we put cannon on a mountain top and shot a cannonball **horizontally** out of it, faster and faster?*"

- From our study of projectile motion, we know that as the **horizontal** velocity the cannonball will travel a further **horizontal** distance, but will take the same amount of time to hit the ground.
  - This is assuming we are on totally flat ground, which would look like Illustration 2 …

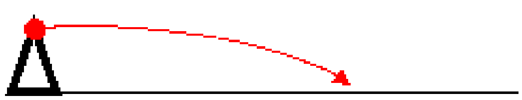

Illustration 2: Cannonball shot horizontally off a cliff, showing flat ground.

Newton knew the Earth is a sphere and therefore has a **curved** surface.

- As the ball travels **horizontally**, the Earth **curves** away from the straight horizontal. On the Earth, the path of the cannonball would still look the same, but the Earth would look different (Illustration 3).

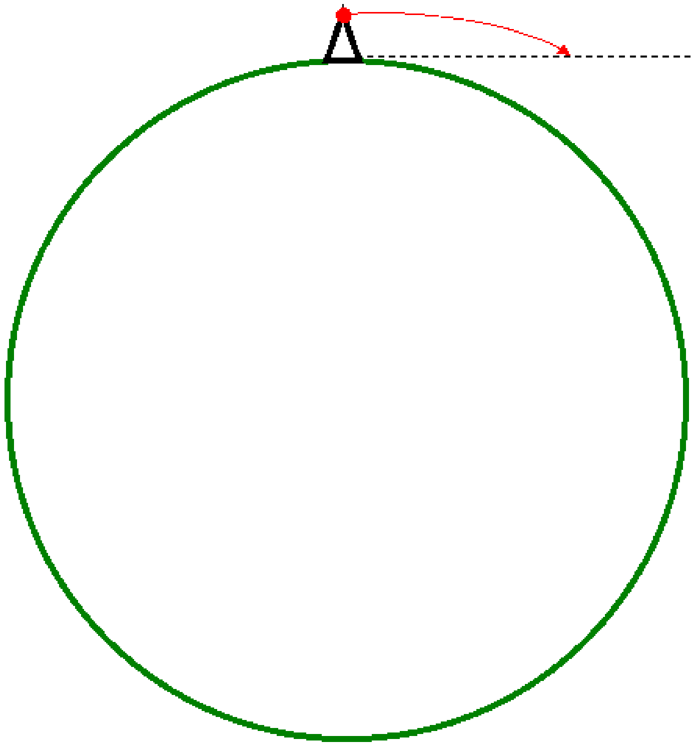

Illustration 3: The path of the cannonball, showing the curve of the Earth.

Notice how the **red line** of the path of the **projectile** matches the **green curve** of the Earth.

- As the **projectile** curves downward, the Earth's **surface** drops away. If this continued, the object would never hit the ground!!!
- In theory, it would go all the way around the Earth and hit the cannon from behind.

Newton wanted to know how fast the object would need to move **horizontally** to follow this orbital path.

- If an object moves roughly 8km **horizontally**, the Earth **curves** down by 4.9m.
    - The reason 4.9m is important is because that is the distance that an object will fall down in one second in regular Earth gravity.
- That way the **curve** of the projectile will exactly match the **curve** of the Earth.

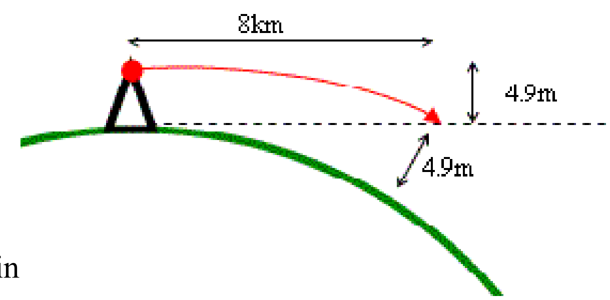

Illustration 4: Curve of the projectile and curve of the Earth.

That would mean that the projectile needs to be shot out of the cannon with an initial horizontal velocity of **8 km/s**.

- That might not seem like much, but look at it this way…

8km/s = 8000m/s = 28 800 km/h!!!

Newton understood that it was physically impossible to build a cannon powerful enough to get something moving this fast..

- He also knew that even if someone figured out how to move something that fast, air resistance would cause it to burn up.
- That is why we have to launch rockets to get into an orbit pretty high up, to get out of most of the Earth's atmosphere.

You can take this idea of satellites a little further to calculate some stuff about either the planet or the satellites.

- Since the satellite is in a more or less circular orbit, the force acting on it must be…

$$F_c=\frac{mv^2}{r}$$

where "m" is the mass of the satellite (since it is the object spinning around).

- We also know that it must be the force of gravity that keeps pulling the satellite in towards the Earth, so…

$$F_g=\frac{GMm}{r^2}$$

where "M" is the mass of whatever is being orbited, like the Earth.

- The force due to gravity is the centripetal force, so we can make the two equal to each other and cancel out…

$$\frac{mv^2}{r}=\frac{GMm}{r^2}$$

$$\frac{v^2}{r}=\frac{GM}{r^2}$$

$$v^2=\frac{GM}{r}$$

$$v=\sqrt{\frac{GM}{r}}$$

- With this formula we can calculate the velocity of a satellite at a particular distance from the centre of the object being orbited (like the Earth).

Example 1: **Determine** how fast a 28.5kg satellite moves if it is orbiting the Earth at an altitude of 120km above the surface.

Remember, the mass of the satellite doesn't matter. Also, you need to measure the distance of the satellite from the centre of the Earth, so look up the values for the radius of the Earth on the back of your data sheet and add it to the altitude in metres. You'll also find the mass of the Earth on the data sheet.

r = 120 000 m + 6.37e6 m = 6.49e6 m
M = 5.97e24 kg

$$v=\sqrt{\frac{GM}{r}}=\sqrt{\frac{6.67\text{e-}11(5.97\text{e}24)}{6.49\text{e}6}}=7832.9929=7.83\text{e}3\,m/s$$

You could also rearrange the formula from above to figure out the mass of the object being orbited.

Example 2: We send a probe to orbit a nearby asteroid and take some pictures of it. The probe enters an orbit that puts it 850m from the centre of the asteroid. If the probe moves at 12m/s, **determine** the mass of the asteroid.

$$v=\sqrt{\frac{GM}{r}}$$

$$M=\frac{rv^2}{G}$$

$$M=\frac{850(12)^2}{6.67\text{e-}11}$$

$$M=1.835082\text{e}15=1.8\text{e}15\,kg$$

We can even do a similar sort of equivalence between centripetal force and gravitational force to get another useful formula.

- This formula is also special in that it allows us to start with **Newton's Universal Law of Gravitation** and finish with **Kepler's Third Law**.
- It is important to realize that in Physics you might take an entirely different route to get to the same theory or formula that was previously discovered.

$$F_c=F_g$$

$$\frac{4\pi^2 mr}{T^2}=\frac{Gm_1m_2}{r^2}$$

$$\frac{4\pi^2 r}{T^2}=\frac{Gm}{r^2}$$

$$\frac{4\pi^2}{Gm}=\frac{T^2}{r^3}$$

The little mass of the satellite on the left cancels the little mass of the satellite on the right.

- Notice how the right hand side is Kepler's Third Law. That must mean that the stuff on the left is equal to Kepler's Constant, K, for whatever is being orbited.
- If necessary, we could solve this formula for any unknown variable. It is written in the form shown above to let you see that it is a different solution to get Kepler's Third Law.

Example 3: Using Newton's version of Kepler's Third Law, **determine** the Kepler's Constant for an object orbiting the Sun.

In the formula we figured out above, the only mass that remains in the formula is the mass of the object being orbited, which is the Sun in this question. Looking up the mass of the Sun on the internet is the only number we need.

$$K = \frac{T^2}{r^3} = \frac{4\pi^2}{Gm}$$

We only need the first and last term.

$$K = \frac{4(3.14)^2}{6.67\text{e-}11(1.99\text{e}30)}$$

$$K = 2.97125809\text{e-}19 = 2.97\text{e-}19\, s^2/m^3$$

If you go back to Lesson 35: Kepler's Three Laws you'll find the value for K for an object orbiting the Sun that we calculated using data on Earth's, Mars', and Jupiter's orbits. The average was 3.99e-29 d^2/m^3. If we convert the value above from seconds squared to days squared, we get...

$$2.97\text{e-}19\frac{s^2}{m^3} x \frac{1\,min^2}{(60\,s)^2} x \frac{1\,h^2}{(60\,min)^2} x \frac{1\,d^2}{(24\,h)^2} = 3.438956122\text{e-}24 = 3.44\text{e-}29\frac{d^2}{m^3}$$

Example 4: **Determine** the altitude of a geostationary satellite around the Earth.

A **geostationary satellite** is a satellite that is always above the exact same location on the Earth at all times. This is useful for satellites like the ones used for TV, since you only have to point your satellite dish at one location in the sky (where the satellite always is) and then just forget about it. Since the geostationary satellite is always at the same location, it must be orbiting around the Earth so that its period exactly matches the period of the Earth, 24 hours.

$$\frac{4\pi^2}{Gm} = \frac{T^2}{r^3}$$

$$r = \sqrt[3]{\frac{T^2 Gm}{4\pi^2}}$$

$$r = \sqrt[3]{\frac{(86400)^2(6.67\text{e-}11)(5.97\text{e}24)}{4(3.14)^2}}$$

$$r = 42\,241\,189 = 4.22\text{e}7\,m$$

We only want the altitude from the Earth's surface, and this answer is the distance from the Earth's centre, so we subtract the radius of the Earth to get our final answer...

$$r = 42\,241\,189 - 6.37\text{e}6 = 35\,871\,188 = 3.59\text{e}7\,m$$

Which means a geostationary satellite is in orbit almost 36 000 km above the surface of the Earth! All this just so you can watch South Park!

Homework

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p279 #1
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