Lesson 33: Horizontal & Vertical Circular Problems

There are a wide variety of questions that you do if you apply your knowledge of circular motion correctly.

- The tough part is figuring out how to set them up.
- You need to figure out two things at the start
  1. Is the circle the object is moving in horizontal or vertical?
  2. What combination of forces are causing the centripetal force?

Horizontal Circle Problems

A horizontal circle is any circle that lies flat parallel to the ground.

- A perfect example is a car going around a regular traffic circle.
- This path can be a complete circle or even just a portion of a circle (as long as you can still measure basics like what the radius would have been if it had been a complete circle).
- If the object is on the ground, it is very common to say that the needed centripetal force is caused by the force of friction with the ground.

Example 1: A construction team is looking at building an exit ramp on the Anthony Henday freeway. They are concerned that cars should be able to go through the turn without skidding off the road even if conditions are bad. **Determine** the minimum radius that can be used for the curve of the turn if a car traveling 110 km/h might be traveling in conditions that cause a coefficient of static friction of only 0.60 between the tires and the road.

- Since the car is moving in a circle there is a centripetal force, caused by the friction between the tires and ground. As the tires push outwards on the ground, the ground pushes inwards on the tires.
- You can see the force of the tires pushing the ground outwards in situations when a car is going around a turn on a gravel road; you can see the gravel being shot outwards from the circle.

$$F_c = F_f$$

$$\frac{mv^2}{r} = \mu F_N$$ ◄— We start by substituting formulas for each side of the relationship.

$$\frac{mv^2}{r} = \mu F_g$$ ◄— Since we are on a flat, level surface F_N is equal to F_g.

$$\frac{mv^2}{r} = \mu mg$$ ◄— This allows us to substitute in $F_g = mg$.

$$\frac{v^2}{r} = \mu g$$ ◄— We can cancel both the “m” on either side, since they it is common to both terms and is found on top on both.

$$r = \frac{v^2}{\mu g}$$ ◄— This allows us to manipulate the formula for “r”, substitute in our values, and solve it.

$$r = \frac{30.6^2}{0.60(9.81)}$$

$$r = 159.082569$$
$$r = 1.6e2\,m$$

This is a huge turn… the diameter would be over 300 m! This would not be reasonable to construct.

Example 2: The easier thing to do than build a huge exit ramp is to simply ask drivers to slow down a bit. This is why exit ramps always have a posted speed limit less than the freeway. The Anthony Henday freeway has exit ramps with a diameter of about 180m. Use this number and a coefficient of static friction of still 0.60 to **determine** the maximum speed that is safe on an exit ramp.

Everything for this problem is the same as Example 1 until you get to what you solve it for, so I'll just skip the first few lines. Don't forget we need to use radius, not diameter.

$$\frac{v^2}{r} = \mu g$$
$$v = \sqrt{r \mu g}$$
$$v = \sqrt{90.0(0.60)(9.81)}$$
$$v = 23 m/s$$

This is about 83 km/h, which is much more reasonable than the speed we used in Example 1.

Vertical Circle Problems

Illustration 1: The London Eye, also known as the Millennium Wheel, is essentially a gigantic ferris wheel.

A vertical circle is any circle that is perpendicular to the ground.

- Although they certainly do not move fast, ferris wheels are a great example of a vertical circle.
- Many high speed roller coasters also make use of at least one vertical circle. These also make a great example for physics questions because of the relationships between the forces involved.

Vertical Circle Considerations

To successfully solve a vertical circle problem you will need to keep a few things in mind. Although this list seems huge, they are all following the regular rules for the directions and signs forces should have.

- For the object to move in a circle, there must be a **centripetal force** acting on it. Examine the particular situation you are looking at and try to decide which force(s) are aligned to add up to give you this special "centre seeking" version of **net force**.
  - If the object is at the top, the **centripetal force** (pointing like it always does, towards the centre) is ***down*** and ***negative***.
  - If the object is at the bottom, the **centripetal force** is pointing ***up*** and ***positive***.
  - The way we can keep track of this in the questions to treat radius kind of like a vector.

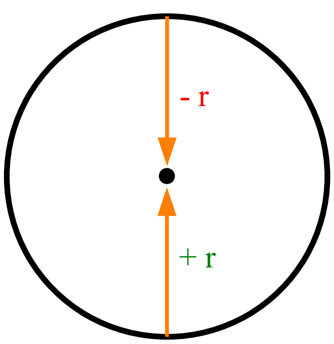

Illustration 2: Treating radius like a vector.

When we are at the top of the loop, radius points **down** to the centre… **- r**

When we are at the bottom of the loop, radius points **up** to the centre… **+ r**

- **Force due to gravity** will often be something you have to consider. It is important to remember that the **force due to gravity** always points straight ***down***.
- In a situation such as a roller coaster, you might be able to show that there is a **normal force** exerted by the surface (like roller coaster tracks) the object is touching.
  - If the object is near the top, the **normal force** points ***down***.
  - If the object is near the bottom the **normal force** points ***up***.
- If the object is not on a surface, it might be spinning on the end of a string or something like that. If this is the case, you will need to take into account the **force due to tension** exerted by the string on the object.
  - If the object is near the top, the **force due to tension** points ***down***.
  - If the object is near the bottom the **force due to tension** points ***up***.

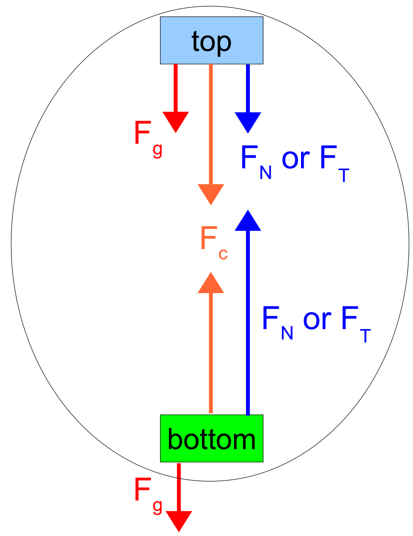

Illustration 3: Direction of forces at top and bottom.

Force	Top of Circle	Bottom of Circle
Centripetal	down	up
Gravity	down	down
Normal	down	up
Tension	down	up

Example 3: A roller coaster is going through a loop that has a radius of 4.80 m. The roller coaster cars have a speed of 13.8 m/s at the top of the loop. During testing and development of the roller coaster, it was determined that the cars and passengers have a combined mass of 4800 kg on an average run. **Determine** the amount of force the track must be designed to withstand at the top in order to keep the cars going around the loop.

There are two forces that will be acting on the cars at the top of the loop.

- The **force due to gravity** will be pulling it down (towards the centre).
- The **normal force** of the tracks pushing against the cars will be pushing the cars down (towards the centre).

These are the two forces that combined will exert the necessary net force, the **centripetal force**, to keep the roller coaster moving in a circle. Notice that the **centripetal force** is also pointing downwards, which we will have to take into account when we substitute into the formula...

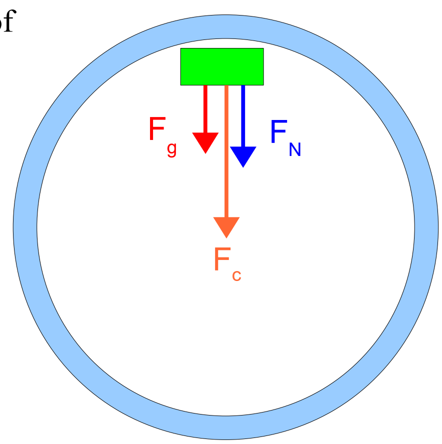

Illustration 4: The car going through the top of the loop.

$$F_c = F_g + F_N$$

We can say that the necessary strength of the roller coaster track is really the **normal force**, since that is the amount of force that the track must exert against the car (just like a table must be strong enough to exert a normal force up against a heavy box placed on the table).

Notice the negative sign we put in for the radius. Since we are at the top, the radius points down to the centre.

$$F_c = F_g + F_N$$
$$F_N = F_c - F_g$$
$$F_N = \frac{mv^2}{r} - mg$$
$$F_N = \frac{4800(13.8)^2}{-4.80} - 4800(-9.81)$$
$$F_N = -143352 = -1.43e5 N$$

The track must be able to exert a force of 1.43e5 N [down] at the top of the loop. If it is not strong enough to exert a force this large, the track will break.

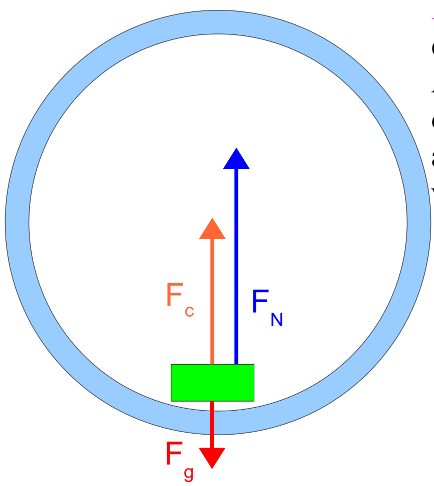

Illustration 5: The car at the bottom of the loop.

Example 4: The track for the roller coaster mentioned in the last example needs to actually be stronger at the bottom of the loop. Although the cars actually speed up as they come down to the bottom of the loop, assume the same velocity, radius, and mass as Example 3 and **determine** the amount of force the track must be able to withstand at the bottom of the loop.

We're going to calculate **normal force** again for the strength of the track, and needs to be quite strong now, since it must support the car against the **force of gravity** ***and*** supply the **centripetal force** needed to keep it moving in a circle.

The magnitudes for $\mathbf{F_c}$ and $\mathbf{F_g}$ stay the same.

We now see $\mathbf{F_c}$ and $\mathbf{F_N}$ point up (positive), but $\mathbf{F_g}$ still points down (negative).

$$F_c = F_g + F_N$$
$$F_N = F_c - F_g$$
$$F_N = \frac{mv^2}{r} - mg$$
$$F_N = \frac{4800(13.8)^2}{4.80} - 4800(-9.81)$$
$$F_N = 237528 = 2.38e5 N$$

Example 5: **Determine** the minimum speed the cars on this roller coaster can move in order to just barely make it through the loop at the top.

Going at the minimum speed means that the cars will just barely make it through the top. The **force due to gravity** will supply all the **centripetal force** needed to keep moving through the circle. There doesn't have to be any **normal force** supplied by the track at all; with **normal force** equal to zero we can just cancel it so we're left with only $\mathbf{F_c}$ and $\mathbf{F_g}$.

$$F_c = F_g + F_N$$
$$F_c = F_g$$

Also, since we're at the top, $\mathbf{F_c}$ and $\mathbf{F_g}$ are both pointing down, so they're both negative. We'll just ignore the negatives on both sides, since they'll just cancel out anyways.

$$F_c = F_g$$
$$\frac{mv^2}{r} = mg$$
$$v = \sqrt{rg}$$
$$v = \sqrt{-4.80(-9.81)}$$
$$v = 6.86\,m/s$$

Here, the negative radius (we are at the top with radius pointing down to the centre) and negative gravity end up with a positive value.

If you need to solve a problem involving an object spinning on the end of the string, you can solve it in a similar way to the questions above.

- The only thing you really need to change is to use **force of tension** in the string instead of **normal force**.

Example 6: A 245 g mass is on the end of a 35 cm long string. **Determine** the tension in the string at the top if the mass is spinning at 5.67 m/s.

$$F_c = F_g + F_T$$
$$F_T = F_c - F_g$$
$$F_T = \frac{mv^2}{r} - mg$$
$$F_T = \frac{0.245(5.67)^2}{-0.35} - 0.245(-9.81)$$
$$F_T = -20.10078 = -20\,N$$

Again, negative radius when at the top of the circle.

Homework

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