Lesson 32: Measuring Circular Motion

Velocity

There should be a way to come up with a basic formula that relates velocity in a circle to some of the basic properties of a circle.

- Let's try starting off with a formula that we know from the beginning of the course.

$$v = \frac{d}{t}$$

- Since we are looking at something going around in a circle, the distance it covers each revolution is equal to the circumference of the circle.

$$C = 2\pi r$$

- We will substitute this into the first formula where the distance the object travels ("d") equals the circumference ("C")...

$$v = \frac{2\pi r}{t}$$

The last thing we need to change is the time "t" on the bottom.

- We're only interested in how much time it takes for the object to go around that circumference once, so what we really need to measure is the **period** of the motion, not its time.
- This gives us a slightly different looking formula...

> DID YOU KNOW?
> The word ***period*** is also used in other sciences, such as the "Periodic Table of the Elements" in chemistry. It is named this way because ***periodically*** the elements repeat the same characteristics.

$$v = \frac{2\pi r}{T}$$

v = velocity (m/s)
π = pi, use 3.14 in your calculations
r = radius of the circle (m)
T = period (s)

There is another way to write the velocity formula if you choose.

- You'll notice that we have period on the bottom.
- This is great, since frequency is just the inverse of period...

$$f = \frac{1}{T}$$

- We can simply replace the period in the original formula with frequency.

$$v = \frac{2\pi r}{T} \quad \text{becomes} \quad v = 2\pi r f$$

Example 1: **Determine** the length of a student's arm if she can swing a pail around five times in a circle at 2.72m/s in 7.5s.

Period is the time it takes to do something once, so...

$$T=\frac{7.5\text{s}}{5\,revs}=1.5\,s$$

Then we can calculate the radius...

$$v=\frac{2\pi r}{T}$$

$$r=\frac{vT}{2\pi}$$

$$r=\frac{(2.72)(1.5)}{2(3.14)}$$

$$r=0.649681529=0.65\text{m}$$

Centripetal Acceleration

We already know that the centripetal acceleration points in towards the centre of the circle.

- Rather than show a tedious proof of the formula for centripetal acceleration, you can probably get along just fine using it.

$$a_c=\frac{v^2}{r}$$

a_c = centripetal acceleration (m/s^2)
v = velocity (m/s)
r = radius of circle (m)

- We only use the magnitude of the velocity of the object. We know that the direction is constantly changing; this is taken into account when we calculate the centripetal acceleration.

Example 2: **Determine** the centripetal acceleration of a person in a car driving at 60km/h in a traffic circle that is 120m across.

Change the velocity into metres per second, and since the measurement of the circle is a diameter, divide it by two.
v = 60km/h = 16.6666 m/s
r = 60.0 m

$$a_c=\frac{v^2}{r}$$

$$a_c=\frac{16.6666^2}{60.0}$$

$$a_c=4.62962963=4.6\,m/s^2$$

You could also combine the velocity formula from earlier with the centripetal acceleration formula

$$v=\frac{2\pi r}{T} \qquad a_c=\frac{v^2}{r}$$

$$a_c=\frac{\left(\frac{2\pi r}{T}\right)^2}{r}$$

$$a_c=\frac{\left(\frac{4\pi^2 r^2}{T^2}\right)}{r}$$

$$a_c=\frac{4\pi^2 r}{T^2}$$

- This formula does not appear on your data sheet, but you might find it useful to do some problems.
  - You should be able to show how you got this formula, so don't just blindly memorize it.

We can also do the same switch as we did with period and frequency in the velocity formula. One small change because we now have squared period...

$$f=\frac{1}{T} \quad \text{becomes} \quad f^2=\frac{1}{T^2}$$

- We can replace the squared period in the original formula with squared frequency.

$$a_c=\frac{4\pi^2 r}{T^2} \quad \text{becomes} \quad a_c=4\pi^2 r f^2$$

Example 3: **Determine** the acceleration of a horse running around a circular race track with a radius of 10 m once every 14 s.

$$a_c=\frac{4\pi^2 r}{T^2}$$

$$a_c=\frac{4(3.14)^2(10)}{14^2}$$

$$a_c=2.012163265=2.0 m/s^2$$

Centripetal Force

We can actually get formulas for calculating centripetal force by quickly combining some simple formulas.

- Keep in mind that the cause or causes of this centripetal force might not be obvious in these formulas. These formulas are simply a way to relate some of the measurable kinematic properties (like velocity) to the centripetal force.

We can easily say that a centripetal acceleration is caused by a centripetal force, so..

$$F_c = m a_c$$

F_c = centripetal force (N)
m = mass (kg)
a_c = centripetal acceleration (m/s^2)

- We can combine this formula with either of the centripetal acceleration formulas. For starters...

$$F_c = m a_c \qquad a_c = \frac{v^2}{r}$$

$$F_c = m\left(\frac{v^2}{r}\right)$$

$$F_c = \frac{m v^2}{r}$$

- We can make another formula with the other centripetal acceleration formula...

$$F_c = m a_c \qquad a_c = \frac{4\pi^2 r}{T^2}$$

$$F_c = m\left(\frac{4\pi^2 r}{T^2}\right)$$

$$F_c = \frac{4\pi^2 m r}{T^2}$$

Just as we did earlier for the centripetal acceleration formula, we can easily substitute frequency for period in the centripetal force formula.

$$F_c = \frac{4\pi^2 m r}{T^2} \quad \text{becomes} \quad F_c = 4\pi^2 m r f^2$$

Example 4: You might have seen movies with an astronaut in training spinning around and around in this big machine to get ready for their flight. The device is called a ***centrifuge***, like the one pictured at right (click on it to go to the NASA website for the Center for Gravitational Biology Research). Let's look at the forces on a 100 kg person's body, and try to relate it back to regular Earth gravity. Keep in mind that just standing on the ground the person in this example would normally weigh 981 N (just use $F_g = mg$).

Illustration 1: A centrifuge.

Determine the centripetal force acting on this 100kg person if he is spun around in a 8.80m radius circle at…

a) 10.0m/s

$$F_c=\frac{mv^2}{r}$$
$$F_c=\frac{100(10.0)^2}{8.80}$$
$$F_c=1136.3636=1.14e3\,N$$

The person is experiencing a force in the centrifuge to get used to the forces he will feel while being launched in a rocket. To say how many gee's the person is feeling, we just need to take this centripetal force divided by the weight of the person.

$$\frac{F_c}{F_g}=\frac{1136.3636}{981}=1.15837=1.16\,gee's$$

This isn't much more than just one gee.

b) 15.0m/s

$$F_c=\frac{mv^2}{r}$$
$$F_c=\frac{100(15.0)^2}{8.80}$$
$$F_c=2556.81818=2.56e3\,N$$

Wow! That's a pretty big increase in force for just a small increase in velocity. Keep in mind that velocity is squared in the formula, so even a small increase can make a big difference. How many gees is the person feeling now?

$$\frac{F_c}{F_g}=\frac{2556.81818}{981}=2.6063=2.61\,gee's$$

You would definitely feel very uncomfortable by this point. This is actually getting closer to the acceleration that astronauts feel going up in the space shuttle.

Example 5: **Determine** the centripetal force acting on a 100 kg man in a 8.80m radius centrifuge if he is spinning at 15 rpm.

Convert rpm to Hz...

15 rpm ÷ 60 = 0.25 Hz

Calculate the centripetal force...

$$F_c=4\pi^2 mrf^2$$
$$F_c=4(3.14)^2(100)(8.8)(0.25)^2$$
$$F_c=2169.112=2.2e3N$$

Homework

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