Lesson 31: Circular Motion

Think of spinning an object on the end of a string. How would you describe it to someone?

- The easiest thing to say is that it moves in the shape of a circle.
    - This means you should be able to measure the **radius** of the circle, and from that calculate things like **circumference**.

For the purpose of the lessons that follow, we will also assume that you keep spinning the bob at the same speed. You don't slow it down or speed it up.

- This means that the bob is rotating at the same *speed* the entire time.
    - We can **not** say that the bob is moving at the same *velocity* the entire time, because at any instant, at any point, the bob is actually changing the direction it is moving.
- Illustration 1 shows a bob being twirled around clockwise on a string. At first, when it is at the top, the direction of the **instantaneous velocity** is to the right.
    - ***The velocity of an object moving in a circle is the tangent line at that point.***
- A moment later the bob is at the next position. Although it is traveling at the same speed (magnitude), the direction it is traveling has changed. It is now pointing down to the right.
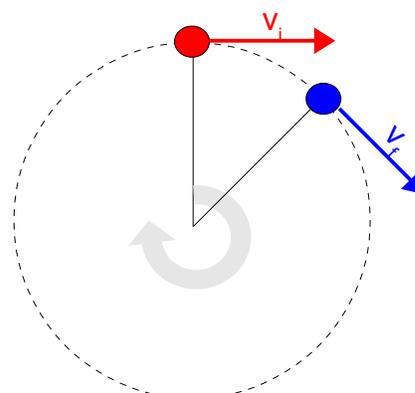

Illustration 1: A bob moving clockwise, showing the instantaneous velocity at two locations.

That means that the *direction* the bob is traveling is constantly changing!

- By definition, this means that the bob is **accelerating** (since acceleration is any change in velocity).
- If it wasn't accelerating, it would be traveling in a straight line (according to Newton's First Law).
- We can get a visual idea of the acceleration as a change in velocity (since Δv is directly related to the acceleration of an object).
    - Since we want $\Delta v = v_f - v_i$, we will use the rules for vectors and show it as $\Delta v = v_f + -v_i$
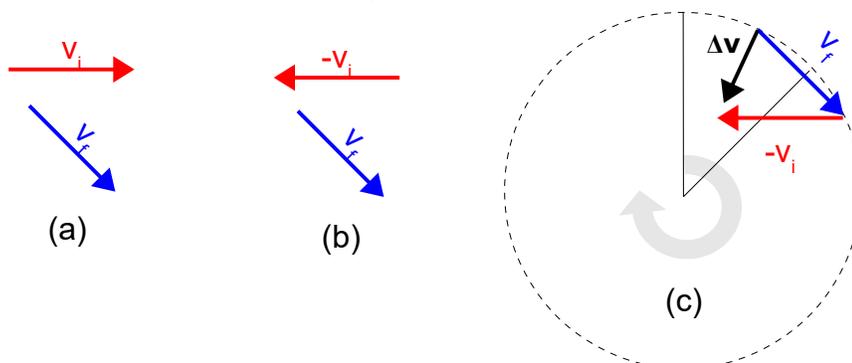

*Illustration 2: (a) The original vectors from Illustration 1 above (b) changed to show -v_i (c) and together on the circle showing **Δv**.*

- Since the black arrow is Δv, it shows us the direction the acceleration is pointing... towards the **centre**. Since the acceleration points towards the centre, we refer to it as **centripetal acceleration**.
  o **Centripetal** comes from Greek, meaning "**centre seeking**"
- The bob is basically being pulled in towards the centre of the circle, which is why it follows a curved path around the centre. It's as though it were seeking the centre of the circle, even though it never gets there.

> Do not confuse this with the word "centrifugal." A person in a car going around in a circle might say that a "centrifugal force" was pushing them across the seat out of the circle. In fact, it is just your body obeying Newton's first law… it is trying to move in a straight line. The car is trying to keep you going in a circle, which is **centripetal**.

By now you should be saying to yourself that if an object going in a circle is **accelerating**, there must be a **force** acting on it.
- This agrees with Newton's Second Law, that a **net force** results in **acceleration**.
  o If there were no forces involved, the object should be moving at a constant speed in a straight line (Newton's First Law), which it isn't.
- Since the acceleration points to the centre, the force must also point to the centre (acceleration is in the same direction as the force causing it). So, just like we talk about **centripetal acceleration**, we also have **centripetal force**.

$$a = \frac{F_{NET}}{m} \quad \text{becomes} \quad a_c = \frac{F_c}{m}$$

> **Warning!**
> **Centripetal force** is not a force of its own (like F_a or F_f). Rather, it is just a new name for net force when we are in the special situation of an object moving in a circle.

For example, you can ask why the Moon orbits around the Earth.
- We know that according the Universal Law of Gravitation, it is the **force due to gravity** of the Earth pulling on the Moon that causes it to orbit.
- If we drew a free body diagram of the Moon, we would only show one force acting on it... F_g.
- To be moving with accelerated motion (in a circle!) there must be a **net force** acting on it.

$$F_{NET} = F_g$$

- But, since it is circular motion, we don't call it **net force**, we call it **centripetal force**.

$$F_c = F_g$$

If you have a hard time seeing that the force must be centripetal (pointing to the centre), think of this example.
- Spin a bob around on the end of a string that you are holding in your hand.
- Your hand is exerting a force on the bob, since otherwise it would go in a straight line.
- No matter when we look at what is happening, your hand is always at the centre of spinning the bob. Since your hand is always at the centre, the only direction your hand can pull the bob is towards the centre.

You might also notice that since the bob is going in a circle, the motion of the bob as it moves around repeats itself over and over again.

- We refer to this as **periodic motion**, because periodically ("every so often") it gets back to where it started and keeps on going.
- Other objects, like pendulums, can also be described as **periodic**, as long as they repeat the same motion over and over again.
- We call the amount of time it takes for the object to go through one revolution (*aka "cycle"*) the **period**.
    - Since **period** is the amount of time to complete one revolution, it has the units **seconds/cycle**. To shorten this, and make it a bit more useful in later formulas, we usually just say the unit for **period** is just **seconds**.
    - That means that we are essentially measuring a time, which we show by choosing a capital "**T**" as the symbol for period in formulas. Just remember that period is a special measurement of time, the amount of time to complete one full revolution or cycle.

Instead of saying how long it takes something to happen, we might sometimes say how often something happens. For example, you might say you go on a vacation every six months, or you go on a vacation twice a year.

- The **period "T"** (time it takes to go through one revolution) is related to its **frequency "f"** (how often something repeats itself).
- **Period** and **frequency** are inverses of each other, so their units must be inverses...

$$T = \text{seconds} / \text{cycle}$$
$$f = \text{cycles} / \text{second}$$
$$T = \frac{1}{f}$$

<div align="right">

T = period (s)
f = frequency (Hz)

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- You will notice that in the metric system, we usually say that the unit for **frequency** is simply Hertz (Hz), which is the same as saying cycles / second.
    - It is common in everyday examples to hear measurements of frequency in different, non-metric, units such as revolutions per minute (rpm).
    - You must convert these measurements into Hertz before using them in questions. Since Hertz is cycles per **second**, and rpm is cycles per **60 seconds**...
    $$rpm \rightarrow Hz$$
    $$\div 60$$
    ...because we want to know how many revolutions there are in just one second, not all 60 seconds.

Example 1: You are thinking of buying a new computer. The hard drive of a computer is a metal disc that spins while data is written or read from it. One of the specifications listed for a computer you are looking at is that the hard drive spins at 7200 rpm.

    a) **Explain** what this means.
    b) **Determine** its frequency in metric units and **explain** what this measurement means.
    c) **Determine** its period.

    a) "7200 rpm" means that the hard drive spins 7200 revolutions per minute. Said another way, if we run the hard drive for one full minute, it will have spun itself around 7200 times.

    b) To convert rpm to Hz we divide by 60, so...
$$7200 \, rpm \div 60 = 120.0 \, Hz$$

This means that the hard drive spins 120.0 times per second.

    c) $T = \dfrac{1}{f} = \dfrac{1}{120.0} = 8.333\text{e-}3 \, s$

Homework

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