Lesson 24: Newton's Second Law (Motion)

To really appreciate Newton's Laws, it sometimes helps to see how they build on each other.

- The First Law describes what will happen if there is ***no net force***.
- The Second Law describes what will happen if there ***is a net force***.

The Second Law (The Law of Motion)

"**When an external, unbalanced force** acts on an object,
the object will **accelerate in the same direction as the force**.
The acceleration varies directly as the force, and **inversely as the mass**."

"**When an external, unbalanced force**…"
We are still talking about these external forces, but now we've added in the idea of "unbalanced".

- Unbalanced just means that there isn't anyone (or anything) pushing against the force hard enough to cancel it out.
    - There is a **net force** acting on the object.

"…**accelerates in the same direction as the force**…"
Some people think this means that the object will move in the same direction as the force… not necessarily!

- The object might be moving to the right, while a force is pushing left.
- That means the object will slow down.
- It's acceleration is in the direction of the force (to the left), but it is still moving to the right.

"**The acceleration varies directly as the force**…"
This just means that if the force increases, the acceleration will increase. If the force decreases, the acceleration decreases.

- This makes sense… push something harder and it will accelerate more! They depend directly on each other.

"…**and inversely as the mass**."
This means that if the mass is bigger, the acceleration is less. If the mass is less, the acceleration is more.

- This makes sense also… if something has less mass, it is easier to make it move faster! They depend inversely on each other.

Mathematically this would be written as…

$$a \alpha F$$

$$a \alpha \frac{1}{m}$$

- The little swimming fish symbol (α) is the Greek letter "alpha" and means "varies as" in math.
- It does ***not*** mean they are equal to each other, but it does show they are related to each other in some way.

- We are showing that acceleration is directly related to force
    - If the net force exerted is increased, the acceleration will also increase.
- Acceleration is inversely related to mass.
    - If the mass increases, the acceleration will decrease.
    - If the mass is bigger, the object will have more inertia, so it will resists changes to its motion more.

When you combine these two relations, you get one of the most basic and important formulas ever discovered in physics.

$$a = \frac{F_{NET}}{m}$$

F_{NET} = net force (**Newtons, N**)
m = mass (**kg**)
a = acceleration (**m/s^2**)

The unit of force is called the Newton.

- It's equivalent to kg m/s^2.
- It was named in honor of the outstanding work that Newton did in physics.
- By definition a one kilogram mass will be accelerated at 1 m/s^2 if a 1 Newton force is applied to it.

Example 1: **Determine** the net force acting on a 5.46kg object if it is accelerated at 17m/s^2.
Since this is the acceleration we observe, the force we can calculate is the net force.

$$a = \frac{F_{NET}}{m}$$
$$F_{NET} = ma$$
$$F_{NET} = (5.46)(17)$$
$$F_{NET} = 92.82 = 93\,N$$

Example 2: **Determine** the acceleration of a 1000 kg car if a 2.5e3 N net force acts on it?

$$a = \frac{F_{NET}}{m} = \frac{2.5e3}{1000} = 2.5\,m/s^2$$

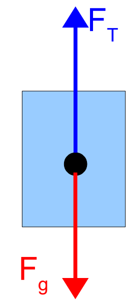
Applications of Newton's Second Law

Usually, we do not deal directly with a net force. We need to remember that typically we have situations where there are several forces acting on an object which together add up tot he net force.

- It is best to start with a free body diagram to get an idea of what is happening with all the different forces.
- Then we can come up with a net force formula based on the forces.
- Be careful in these examples that you do not mix horizontal and vertical forces (when they both happen in the same question, like example 6).

Example 3: A 3.70e3 kg elevator is being raised by a cable that exerts a 4.00e4 N force upwards. **Determine** the acceleration of the elevator.

- There are only two forces acting on the elevator, the force of tension in the cable pulling it up and the force of gravity acting down.
- It is the net force that causes the acceleration of the elevator, so we're basically solving for that. We can substitute in F_{NET} = ma
- Remember to use negative for down and positive for up.



Drawing 1: Free body diagram of elevator.

$$F_{NET} = F_g + F_T$$
$$ma = mg + F_T$$
$$a = \frac{mg + F_T}{m}$$
$$a = \frac{3.70e3(-9.81) + 4.00e4}{3.70e3}$$
$$a = \frac{-36297 + 4.00e4}{3.70e3}$$
$$a = \frac{3703}{3.70e3}$$
$$a = 1.0008108 = 1.00\, m/s^2$$

The acceleration of the elevator is 1.00 m/s^2 [upwards].

Example 4: **Determine** the acceleration of the elevator from Example 3 if the tension in the cable is only 3.30e4 N.

$$F_{NET} = F_g + F_T$$
$$ma = mg + F_T$$
$$a = \frac{mg + F_T}{m}$$
$$a = \frac{mg + F_T}{m}$$
$$a = \frac{3.70e3(-9.81) + 3.30e4}{3.70e3}$$
$$a = \frac{-36297 + 3.30e4}{3.70e3}$$
$$a = \frac{-3297}{3.70e3}$$
$$a = -0.8910810811 = -0.891\, m/s^2$$

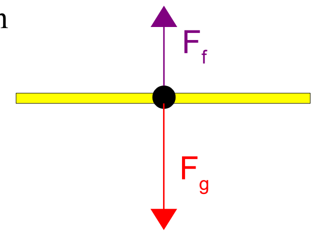
The acceleration of the elevator is now 0.891 m/s^2 [down].

Example 5: A 0.0500 g piece of paper is dropped. If it falls with an acceleration of 0.390 m/s² [down] **determine** the friction acting on the paper.

- The frictional force must be pointing upwards since it will resist the motion of the paper falling down.
- Don't forget to turn the mass into kilograms.

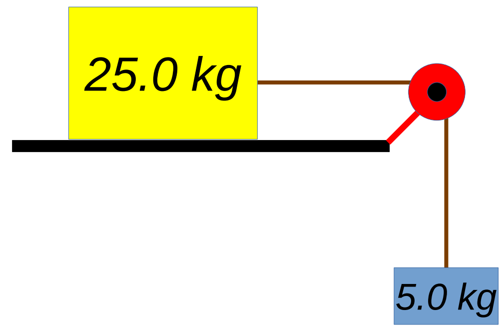
$$F_{NET}=F_g+F_f$$
$$ma=mg+F_f$$
$$ma-mg=F_f$$
$$F_f=ma-mg$$
$$F_f=m(a-g)$$
$$F_f=5.00\text{e-}5(-0.390-(-9.81))$$
$$F_f=4.71\text{e-}4\,N$$

The friction acting on the paper is 4.71e-4 N [up].



Drawing 2: Free body of falling sheet of paper.

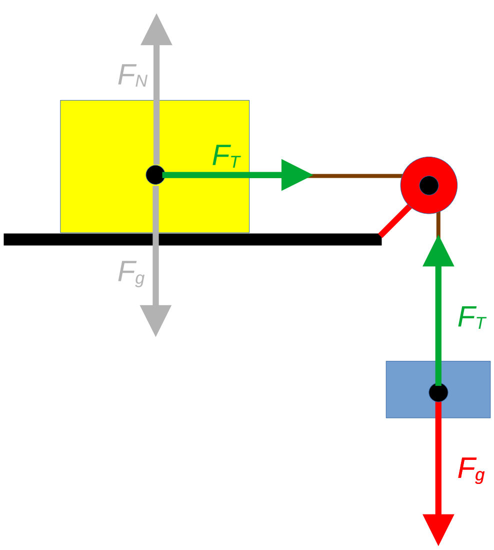
Example 6: Two boxes are attached by a thin, strong, zero-mass wire as shown in *Drawing 3*. The 25.0 kg yellow box is sitting on a frictionless table, with the wire going over a frictionless pulley to the 5.0 kg blue mass hanging in the air. At the start, someone is holding the 5.0 kg mass motionless where it is. If the person releases the 5.0 kg, **determine** the acceleration of the system.



Drawing 3: Pulley System

There are a few things to keep in mind for these types of questions...

- We were told everything is frictionless, so we just won't do anything with friction (makes it easier).
- Pulleys can do the magical act of changing the direction of forces, as you'll see in the solution of this question.
- The force of gravity acting on the 5.0kg mass is the only force acting on the system that can make anything accelerate.
- We keep referring to this as a "system" because with the two masses attached by a wire, they will act together like one 30.0 kg mass.

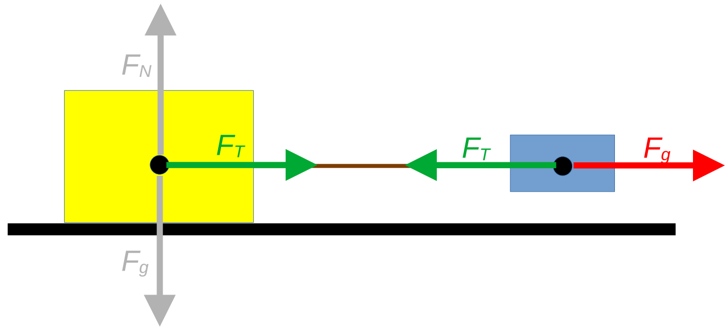


Drawing 4: Free body diagrams

If we sketch the free body diagram of what is happening in this situation we will get *Diagram 4*.

- For simplicity of just having one diagram to look at, we are just showing it all at once.
- We will be ignoring the **vertical forces** acting on the 25.0kg mass since they just cancel out.
- It is important to notice that the two **forces of tension** are equal (Newton's Third Law), with the yellow box being pulled across the table because the blue box is pulling on it. At the same time, the blue box feels the yellow box pulling up on it.
  - Depending on how the question is set up, we could have been asked to calculate this tension.
- The **force of gravity** pulling down on only the blue box is the whole reason why this system will move.

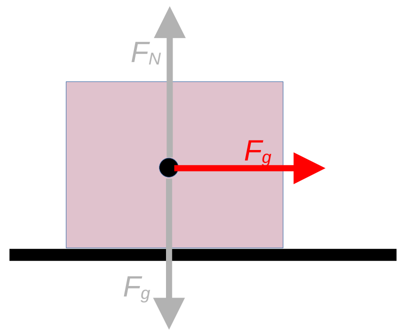
To solve the question, we can “roll” the 5.0 kg blue box up over the pulley mentally, changing the direction of its forces (but not their magnitudes)...



Drawing 5: Up and over the pulley

Finally, we treat it as one big 30.0kg mass (since they are attached), meaning we no longer need to consider the force of tension.

- Remember, the **force of gravity** is really just created by the 5.0 kg mass being acted on by gravity, so it is the only mass that will appear in that part of the calculation.
- The entire 30.0 kg of mass needs to be accelerated by this force, so we use 30.0 kg when doing the part of the calculation for the acceleration.



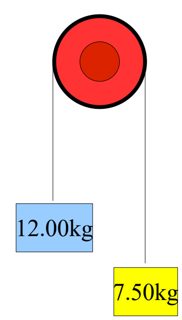
Drawing 6: One big mass

$$F_{NET} = F_g$$
$$ma = mg$$
$$30.0a = 5.0(9.81)$$
$$30.0a = 49.05$$
$$a = 1.635 = 1.6\,m/s^2$$

We would see the yellow box accelerate to the right at 1.6 m/s^2, and the blue box accelerate downwards at 1.6 m/s^2.

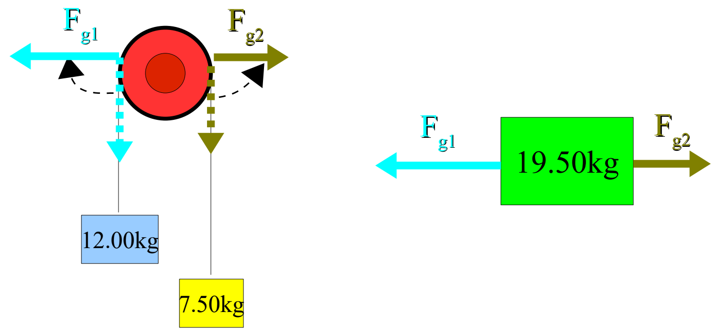
Example 7: “Atwood's Pulley” is a special pulley problem involving two unequal masses hanging from a wire over a pulley as shown in *Drawing 7*. Since one of the masses is heavier, the whole thing will move so that the heavier mass moves down and the lighter mass moves up. If mass one is 12.00 kg and mass two is 7.50 kg, **determine** the acceleration of each mass.

- Since we are dealing with two masses that are attached, we will do the same thing as Example 6 and add the masses.
- We will need to be careful, since the net force is pulling the blue box down and the yellow box up, but is made up of two forces due to gravity pulling down on different sides.



Drawing 7: Atwood's Pulley.

To simplify this, we will take our single mass and pretend that one force due to gravity is pulling it to the left, while the other force due to gravity is pulling it to the right.



Drawing 8: Rotate the two forces due to gravity horizontally, and show them acting on the combined mass.

Mass One

$$F_{g1} = mg$$
$$F_{g1} = 12.00(-9.81)$$
$$F_{g1} = -117.72\,N$$

Pulls to the left with 117.72N.

Mass Two

$$F_{g2} = mg$$
$$F_{g2} = 7.50(9.81)$$
$$F_{g2} = 73.575\,N$$

Pulls to the right with 73.575N.

$$F_{NET} = F_{g1} + F_{g2}$$
$$F_{NET} = -117.72 + 73.575$$
$$F_{NET} = -44.145$$
$$F_{NET} = -44.15\,N$$

$$F_{NET} = ma$$
$$a = \frac{F_{NET}}{m} = \frac{-44.15}{19.50}$$
$$a = -2.26\,m/s^2$$

The minus sign, based on Drawing 6, just shows that mass one will drop as mass two rises. Both will have an acceleration of 2.26 m/s^2.

When doing questions that involve a pulley, just remember that a pulley kind of changes the direction a force is acting in. That's why we can change the directions some of the forces are pointing in these questions, even though we typically can not change the direction of vectors.

Homework

p149 #1,2
p150 #1
p152 #2
p158 #5, 8, 9