

Lesson 15: Solving Vector Problems in Two Dimensions

We can now start to solve problems involving vectors in 2D.

- We will use all the ideas we've been building up as we've been studying vectors to be able to solve these questions.
- The majority of questions you will work on will involve two **non-collinear** (*not in a straight line*) vectors that will become part of a right-angle triangle. If there are more than two vectors, you will probably be able to use a trick or two that will allow you to get a triangle out of them.

Since the triangles will be right-angled, you will be able to use a bunch of your basic math skills.

- Just use your regular trig (SOH CAH TOA) and Pythagoras ($c^2 = a^2 + b^2$).
- Usually you'll want to be thinking about physics as you set up your diagram (so that you get everything pointing head-to-tail and stuff) and then switch over to doing it like any math trig problem.

Example 1: On a hot summer day a person goes for a walk to see if they can find a 7-Eleven to buy a Slurpee. He first walks 3.5 km [N], then 4.2 km [E], and finally 1.4 km [S] before getting to the 7-Eleven. Oh, thank heaven! **Determine** the displacement of the person.

A quick sketch will help us organize things so we can get to work.

This shows the vectors being added head to tail, and the resultant that the person actually moved, start to finish. Now consider this; at the end when the person walked 1.4 km [S] he was basically undoing some of the original 3.5 km [N] that he had originally walked. We can add these vectors in any order we want, so let's just rearrange things so that we add the first and the last vectors together (since they are collinear), then take care of the other one.

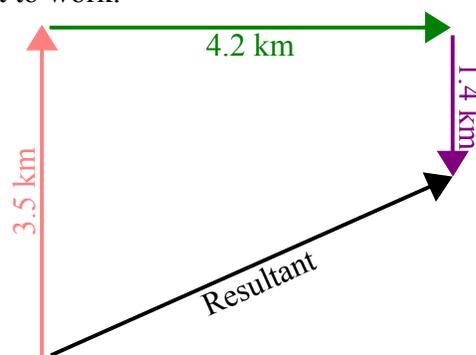


Illustration 1: Adding three vectors to get the resultant.

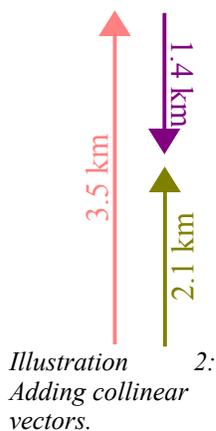


Illustration 2: Adding collinear vectors.

Assuming North is positive and South is negative...

$$+3.5 \text{ km} + -1.4 \text{ km} = +2.1 \text{ km}$$

This basically shows that by walking North and then South, the person overall moved 2.1 km [N]. Now we can use just this one vector moving North along with the vector pointing East and draw one simple triangle diagram to get our answer.

$$c^2 = a^2 + b^2$$

$$c^2 = 2.1^2 + 4.2^2$$

$$c = 4.695743$$

$$c = 4.7 \text{ km}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan \theta = \frac{4.2}{2.1}$$

$$\theta = 63.43495^\circ$$

$$\theta = 63^\circ$$

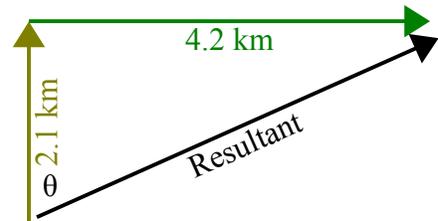


Illustration 3: Adding two vectors to get a resultant.

The person walked a displacement of 4.7 km [N63°E].

You can also break a resultant apart to get components if you need to.

- On its own this might not be very useful (like **Example 2** shows).
- Sometimes it can allow you to do questions that are more complex (as **Example 3** shows).

Example 2: A wagon is being pulled by a rope that makes a 25° angle with the ground. The person is pulling with a force of 103 N along the rope. **Determine** the horizontal and vertical components of the vector.

We could just as well be asked to find the x-component and y-component... it's the same thing. It would probably be helpful to draw a quick sketch to help organize your thoughts. Then you can start using trig to find the components.

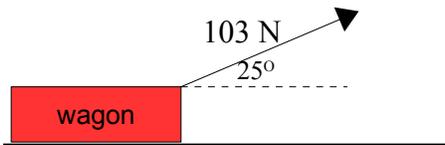


Illustration 4: Pulling a wagon with a rope.

We want to know the components of the 103 N vector, so we basically need to draw a triangle and then solve for the unknown sides. The triangle will be drawn using the force vector from above along with horizontal and vertical components.

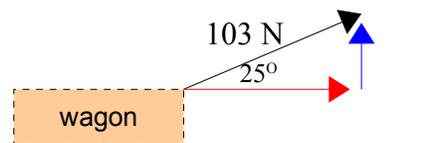


Illustration 5: Triangle with unknown sides.

Now solve for the two unknown sides of the triangle.

$$\text{Horizontal}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\text{adj} = \cos \theta (\text{hyp})$$

$$\text{adj} = \cos 25^\circ (103)$$

$$\text{adj} = 93.3497021 = 93 \text{ N}$$

$$\text{Vertical}$$

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\text{opp} = \sin \theta (\text{hyp})$$

$$\text{opp} = \sin 25^\circ (103)$$

$$\text{opp} = 43.529681 = 44 \text{ N}$$

Example 3: A plane flies 34 km [N30°W] and after a brief stopover flies 58 km [N40°E]. **Determine**

the plane's displacement.

Draw a very careful diagram for questions like this one. Make sure that you carefully draw in the angles and properly show the two vectors being added head-to-tail.

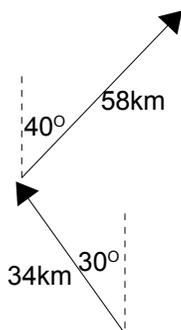


Illustration 6: Two vectors added head-to-tail.

We can't add the vectors like this, so we need to use a way to simplify things. The easiest thing to do is take each of the vectors individually and break it into its components.

Once you have two x and two y components, you can add them **x to x** and **y to y**.

Then you will have one set of x and y components which can be added to give you your resultant.

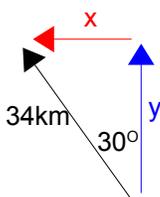


Illustration 7: First vector broken into components.

x - component

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\text{opp} = \sin \theta (\text{hyp})$$

$$\text{opp} = \sin 30^\circ (34)$$

$$\text{opp} = 17 \text{ km}$$

y - component

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\text{adj} = \cos \theta (\text{hyp})$$

$$\text{adj} = \cos 30^\circ (34)$$

$$\text{adj} = 29.444864 \text{ km}$$

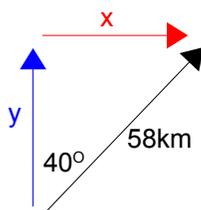


Illustration 8: Second vector broken into components.

x - component

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\text{opp} = \sin \theta (\text{hyp})$$

$$\text{opp} = \sin 40^\circ (58)$$

$$\text{opp} = 37.2816814 \text{ km}$$

y - component

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\text{adj} = \cos \theta (\text{hyp})$$

$$\text{adj} = \cos 40^\circ (58)$$

$$\text{adj} = 44.4305777 \text{ km}$$

Now here's the nifty part. Both of the y components are pointing up, so we'll call both of them positive. When we add them we get...

$$29.444864 \text{ km} + 44.4305777 \text{ km} = 73.8754417 \text{ km}$$

This is positive so it is also pointing up.

The x components are a little different. The first one points to the left so we will call it negative. The second x component pointing to the right we will call positive. This gives us...

$$-17 \text{ km} + 37.2816814 \text{ km} = 20.2816814 \text{ km}$$

This is positive so it is pointing to the right.

Draw a new diagram made up of just these two new vectors and find the resultant.

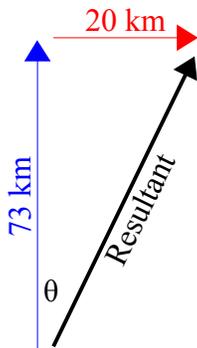


Illustration 9: Final resultant.

$$c^2 = a^2 + b^2$$

$$c^2 = 73.8754417^2 + 20.2816814^2$$

$$c = 76.6089256 = 77 \text{ km}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan \theta = \frac{20.2816814}{73.8754417}$$

$$\theta = 15.351686^\circ = 15^\circ$$

The plane's displacement is 77 km [N15°E].

Homework

p82 #2
p84 #1,3
p88 #2