

# Lesson 13: Vectors in One Dimension

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Up to this point we have been focusing on the number crunching sort of questions you can do in physics.

- In this chapter the focus will start to be shifted toward more complicated problems that might not always be solved by just “plugging numbers into a formula.”
- For this reason, we will start to use **vector diagrams** as a way to organize our information and to help us solve our problems.
  - This is more than just listing givens... it really is a necessary step to be able to solve the problem.
- As we start to use these diagrams, keep in mind that we are drawing diagrams that truly represent the motion of the object.
- *Mastering vector diagrams is **critical** to your success in all physics courses you will take in the future!*

As we learned back in **Lesson 8**, just about anything you measure in Physics can be divided into two categories: **scalars** and **vectors**.

**Scalars:** Any measurement that is given as a single number, and nothing else. It has **magnitude**, but no direction.

**Vectors:** A measurement that is given as a number and a direction. It has **magnitude** and **direction**.

We often use arrows to represent vectors. In fact, for the rest of the course you should see them as being interchangeable; an arrow in a diagram is a vector.

- When you have several of these vectors drawn together, you have a vector diagram.
- Although vector diagrams are drawn for different reasons in different kinds of problems, the rules that govern how they are drawn are always the same.

## Vector Drawing Rules

1. **The vector is drawn pointing in the direction of the vector.** This is probably the key feature of what makes a vector a vector... *direction*. If an object is moving East, you better make sure that the arrow points East. Always remember that when a direction is written down with the magnitude of a measurement, the direction must appear in square brackets.
2. **The length of a vector is proportional to the magnitude of the measurement.** This just means that the bigger your measurement, the bigger your vector. If I wanted to show you a vector for a car moving at 10 km/h [East], and another one moving at 20 km/h [East], the second vector would be twice as big.
3. A vector can be picked up and moved around in a vector diagram, as long as when you place it in its new location it is still **the same size and pointing in the same direction**. This is actually a big deal, since we will see later that moving vectors around to arrange them is critical to solving problems.
4. There are very specific rules for how vectors can touch each other, most of which will make more sense by the end of this lesson. Vectors must touch **head-to-tail** (aka tip-to-tail) when you are **adding** them. Only **resultants** can touch **head-to-head** and **tail-to-tail**. Like I said, more on this soon.

If you are extremely careful, you can even use the rules to draw all your vector diagrams to scale (like 1 cm = 1 m) and solve them by measuring stuff without using physics or math formulas.

- I would caution you that calculations will usually give you more accurate answers than a diagram.
  - This is simply because any small errors in your diagram measurements will be magnified by the scale you used.

## Adding Vectors In One Dimension

One dimensional vector diagrams are the easiest ones to solve.

- We refer to vectors that line up with each other as **collinear** (“together linear”) vectors.
- The only thing you have to really watch out for is how you touch the vectors to each other in the diagram.
- To add vectors, the vectors must touch with the head of one (the pointy tip) touching the tail (nothing there) of the next vector.

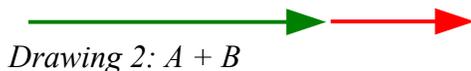
**Example 1:** Sketch a diagram that shows how you would add the following two vectors, **A** and **B**.



*Drawing 1: The original vectors A and B*

In the text book they call vectors that point in the same direction (like these two) **collinear**.

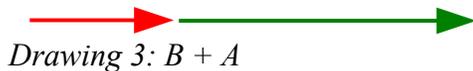
Right now **A** and **B** are not touching head-to-tail, so we will have to rearrange them. Keep in mind that I can pick up and move around the vectors as long as I make sure that when they are plunked back down they are still pointing the same direction and the same size.



*Drawing 2: A + B*

Drawing 2 shows **A + B**. Notice how the vectors individually are still exactly the same, but they have been rearranged so that **A** is first and then touches **B** head-to-tail.

If I wanted to draw **B + A** instead, it would look like Drawing 3.



*Drawing 3: B + A*

You'll notice that it doesn't matter in what order I add the two vectors. **A + B = B + A**.

The idea that I get the same result when I add the vectors shown above is a very important one in Physics.

- It would be like if I told you that I was planning to walk 4 km East, and then another 6 km East. It would give me the same result (10 km East) as I would get by first walking 6 km and then 4 km.
- Since the end result is 10 km East, I would say my **resultant** is 10 km [E].

- A **resultant** is the sum total of two or more vectors added. It shows you what you would get as an end result of the other vectors put together.

**Example 2:** Sketch the resultant of the addition of the two vectors in Example 1.

Since we already figured out that  $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$  we can draw the resultant for either combination and get the same thing. We'll just go ahead and draw it for  $\mathbf{A} + \mathbf{B}$ .



*Drawing 4: Resultant of  $A + B$ .*

The resultant is just another vector drawn as an arrow. The difference is that the **resultant** touches **tail-to-tail** and **head-to-head**. This is because the resultant has to start at the beginning (the tail of  $\mathbf{A}$ ) and finish at the end (the head of  $\mathbf{B}$ ). That's the only way for the resultant to show that it has the same overall result as the two original vectors.

There is an “old-school” way to remember what's happening with these vectors. It involves going back to my younger years playing the video game Pac Man.

- Let's assume that the original vectors  $\mathbf{A}$  and  $\mathbf{B}$  are showing where Pac Man was moving. Starting at the tail of  $\mathbf{A}$ , Pac Man moves to the right. Then he continues along  $\mathbf{B}$  a little more to the right.
- Overall, Pac Man has moved to the right a whole bunch... that's what the resultant shows us. Pac Man can follow the two individual vectors  $\mathbf{A}$  and  $\mathbf{B}$  to get from the beginning to the end, or he can just follow the one resultant and end up in the same place from the same beginning.

## Subtracting Vectors in One Dimension

This is where things get a bit more interesting.

- What we need to remember here is that in Physics a *negative* sign simply means “*in the opposite direction.*”
- We can take  $\mathbf{A} - \mathbf{B}$  and simply change it into  $\mathbf{A} + -\mathbf{B}$ .
  - The negative sign on the  $\mathbf{B}$  just means that we will need to take the original vector  $\mathbf{B}$  and point it in exactly the opposite direction ( $180^\circ$  from where it's pointing originally).
  - Then we will simply add them just like we did in the previous diagrams (touching head-to-tail of course!) to get our resultant.
- The reason you may have to do subtraction of vectors is because some physics formulas require you to subtract vectors. For example,  $\Delta v = v_f - v_i$ .

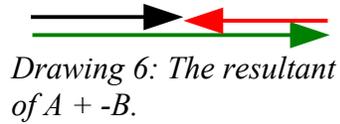
**Example 3:** Sketch a vector diagram of  $\mathbf{A} - \mathbf{B}$ .

The most important thing to remember is that  $\mathbf{A} - \mathbf{B}$  is equal to  $\mathbf{A} + -\mathbf{B}$ . So, all we need to do is take the original vector for  $\mathbf{B}$  and spin it around so it points in the opposite direction.



*Drawing 5: Vectors  $A$  and  $-B$ .*

Now we just add them head-to-tail and draw in our resultant.



Using the Pac Man analogy, You can see that if Pac Man moved to the right a lot (**A**), and then moved back the left a bit (**B**), then it would be the same if he'd moved to the right just a bit (**resultant**).

Be careful since subtraction is *not* commutative (that just means that  $A - B \neq B - A$ ).

## **Homework**

p73 #1

p74 #1, 2

p75 #1, 8