Lesson 11: The Other Formulas

There are several other formulas that are very useful when the acceleration is uniform.

- Do not use these equations if the acceleration is changing! The acceleration must be constant.
- These other formulas are based on combinations of the basic velocity and acceleration formulas, as well as interpreting graphs.
- Although the formulas are shown here in a particular numbered order, you do *not* need to identify them this way.

Formula 1

The first formula is based on knowing information about displacement, final and initial velocities, and time.

$$d = \left(\frac{v_f + v_i}{2}\right) t$$

In your text book, this formula is written slightly differently as $d = 1/2\,(v_i + v_f)\,t$. It is still the exact same formula.

- At first this may seem to be an odd acceleration formula, since acceleration does not appear in the formula as a variable.
  - Notice that there are two velocities, v_f and v_i, so we know that there must be acceleration.
  - Only use this formula when you know for certain that the object has been going through a constant acceleration, even though "a" doesn't appear in the formula.

Example 1: **Determine** how far a vehicle moved if it started at 12 m/s and accelerated up to 28 m/s in a time of 34s.

$$d = \left(\frac{v_f + v_i}{2}\right) t$$

$$d = \left(\frac{28 + 12}{2}\right) 34$$

$$d = 680 = 6.8\text{e}2\, m$$

Formula 2

We will do problems where we have information about displacement, initial velocity, time, and acceleration. The formula for these situations is...

$$d = v_i t + ½\, a t^2$$

- Be careful with this formula. Only the time is squared in the last term, not acceleration and time.
- As a bonus, a lot of the time v_i will be zero, which cancels out the first term and leaves you with…

$$d = ½\, a t^2$$

Example 2: Occasionally the US Air Force calls me in to test fly their "birds". A few weeks back I was flying along in my F-22 at 97m/s when I decide to kick in the afterburners for 12.3s. If the afterburners can generate enough thrust to accelerate the F-22 at 26m/s^2, **determine** how far I travelled during that time.

$d = v_i t + \frac{1}{2} at^2$
$= (97)(12.3) + (0.5)(26)(12.3)^2$
$d = 3159.87 = 3.2e3$ m

Example 3: I am in a F-22 that is on the runway. From rest, I accelerate the plane at 3.9m/s^2 for 14.5s to reach take off velocity. **Determine** how long the runway must be.

This is an example of a question where the initial velocity is zero (since I'm starting from rest), so…

$d = v_i t + \frac{1}{2} at^2$
$d = \frac{1}{2} at^2$
$= (0.5)(3.9)(14.5)^2$
$d = 409.9875 = 4.1e2$ m

Formula 3

There is a formula related to formula 2 that can be used when we know the final velocity instead of the initial.

$$d = v_f t - \frac{1}{2} at^2$$

- Notice that the differences are final instead of initial velocity, and the minus sign instead of addition.
- Otherwise, this formula is used the same way as formula 2.

Example 4: Now the F-22 is coming in for a landing. If the runway is 2500 m long, and the plane uses the whole length to come to a stop in 43s, **determine** the acceleration of the plane.

Big thing to note here is that the plane coming to a stop means that the final velocity is zero.

$$d = v_f t - \frac{1}{2} at^2$$
$$d = -\frac{1}{2} a t^2$$
$$a = \frac{-2d}{t^2}$$
$$a = \frac{-2(2500)}{43^2}$$
$$a = -2.7041644132 = -2.7\, m/s^2$$

Notice the negative sign, indicating that the brakes are slowing down the plane.

Example 5: A car drives 83m while accelerating at 2.4m/s^2 for 4.9s. Determine the final velocity of the car.

We're going to have to manipulate the formula to solve for v_f. Keep in mind that you may have to manipulate any of the formulas we are looking at.

$$d = v_f t - 1/2\, at^2$$
$$v_f = \frac{d + 0.5\, a\, t^2}{t}$$
$$v_f = \frac{83 + (0.5)(2.4)(4.9)^2}{4.9}$$
$$v_f = 22.81878 = 23\, m/s$$

Formula 4

Another very useful formula is the following…

$$v_f^2 = v_i^2 + 2ad$$

- Very handy when you have a question with both velocities, acceleration, and displacement.
- Don't forget to do the square root at the very end if you are solving for a velocity, as the following example shows…

Example 6: **Determine** the final velocity of a car that starts at 22 m/s and accelerates at 3.78 m/s^2 for a distance of 45 m.

$$v_f^{\,2} = v_i^{\,2} + 2ad$$
$$v_f^{\,2} = 22^2 + 2(3.78)(45)$$
$$v_f^{\,2} = 824.2$$
$$v_f = 28.70888 = 29\, m/s$$

Many people forget that in the last step you need to square root in order to get the velocity instead of the velocity *squared*!

How to Choose the Right Formula!

So, how do you figure out which formula to use for a particular problem?

- As you look back through the formulas, you'll see that of the five basic things we measure about the motion of an object (v_f , v_i , a , t , and d) , each formula only has four.
    - To choose the correct formula, figure out the one thing that you are ***not*** given and ***not*** asked for in the question. Choose the one formula that does ***not*** have that variable.
- The following table may help.

Formula	a	v_f	v_i	d	t
$a=\frac{v_f-v_i}{t}$	*	*	*	X	*
$d=\left(\frac{v_f+v_i}{2}\right)t$	X	*	*	*	*
$d = v_it + ½ at^2$	*	X	*	*	*
$d = v_ft - ½ at^2$	*	*	X	*	*
$v_f^2 = v_i^2 + 2ad$	*	*	*	*	X

For example, let's say I had a question where I am given acceleration, displacement, and time, and asked to find initial velocity.

- The only thing I wasn't given, and I wasn't asked for, is final velocity.
- The only formula that does not have final velocity is $d = v_it + ½ at^2$. This is the formula to use.

Remember that for all of these formulas, you may be required to manipulate the formula to find the answer you are looking for.

- Always follow the rule of finding the formula that has all the knowns and unknown that you have.
- Write down the original formula as it appears on the data sheet.
- Then manipulate it for your unknown, and solve.

There can be special situations where, ***in two separate parts***, we need to use uniform and accelerated motion formulas…

Example 7: You are riding in a bus that is moving at 50 km/h [forwards]. The light turns red up ahead. The bus driver takes 1.8 s to react and move their foot to the brake pedal. When they press on the brakes, the bus slows to a stop with an acceleration of 1.92 m/s^2 [backwards]. **Determine** the total distance the bus travelled.

Uniform Motion (Before Braking)

$$v=\frac{d}{t}$$
$$d=vt$$
$$d=13.9(1.8)$$
$$d=25\,m$$

Accelerated motion (While Braking)

$$v_f^2=v_i^2+2ad$$
$$d=\frac{v_f^2-v_i^2}{2a}$$
$$d=\frac{0^2-13.9^2}{2(-1.92)}$$
$$d=50\,m$$

Total distance travelled = 25 + 50 = **75 m**

Homework

p.48 #1,2
p.50 #1,2
p.51 #1,2
p.52 #1,2
p.53 all odd