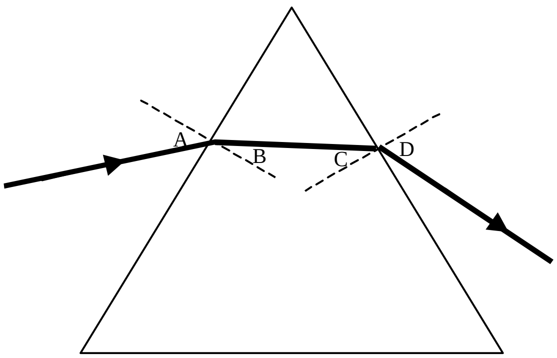
Deviation in a Prism and Displacement through Flat Glass

The purpose of this handout is to help you do some of the questions you are finding at the end of Chapter 17, specifically page 364 #14 and page 365 #15.

Deviation in a prism

The first question involves light that is traveling through an equilateral triangular prism (all three angles of the triangle are 60°) as shown below:



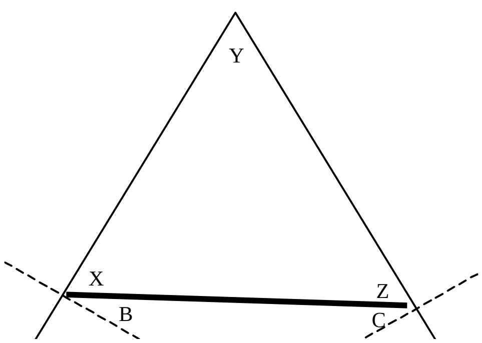
Assume for this example that the prism is made out of glass (n = 1.5) and it is surrounded by air (n = 1.0).

Notice that as the beam of light enters the glass it bends *towards* the normal. It then travels through the glass and leaves the other side bending *away* from the normal. The problem is, what angle do you use at "C" to figure out the angle "D"?

To show you how to figure this out, let's first just look at the angle "A" and "B"… forget about the rest of the prism for now and just treat it like a regular Snell's Law question. Assume the light hits the glass at a 35° angle.

$n_{air} = 1.0$ $n_{glass} = 1.5$ Angle A = 35° Angle B = ?	$(\sin A)\, n_{air} = (\sin B)\, n_{glass}$ $\dfrac{(\sin 35°)1.0}{1.5} = \sin B$	B = 22.5°

So angle "B" is 22.5°, but what is angle "C"? Well, first I'll show you the math behind it, then I'll give you the quick trick! For this part I'll zoom into the top part of the prism…



- You know B = 22.5° and you also know that the normal line drawn there is 90°. This means that angle X = (90° - 22.5°) = 67.5°.
- Because this is an equalateral triangle, angle Y must be 60°.
- The sum of the angles of any triangle must equal 180°, so X + Y + Z = 180°, which means…
  67.5° + 60° + Z = 180° ➔ Z =52.5°
- Just like above, Z + C = 90° so,
  52.5° + C = 90° ➔ C = 37.5°

I know this might not be easy, but read through these steps a couple times while looking at the diagram.

Are you ready for the quick trick? Take a look at the values you have for "B" and "C"… do you notice anything? Well, here it is…

$$\boxed{\mathbf{B + C = 60°}}$$

That's it! So once you did the calculations in the box above and you know "B" is 22.5°, all you have to do is subtract it from 60° and you'll have the angle for "C".

Let's finish off the question.

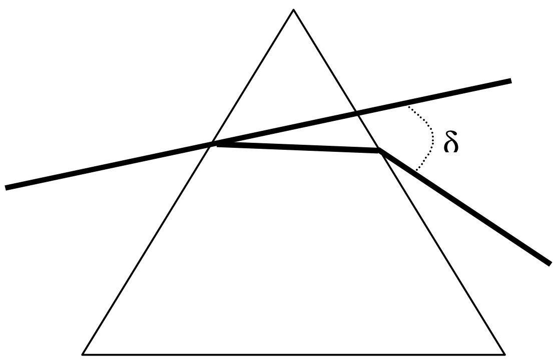
$n_{air} = 1.0$
$n_{glass} = 1.5$
Angle $C = 37.5°$
Angle $D = ?$

$$(\sin C)\, n_{glass} = (\sin D)\, n_{air}$$

$$\frac{(\sin 37.5°)1.5}{1.0} = \sin D$$

$D = 66°$

There is one last curve ball I can throw at you. Look at the original diagram and imagine the path the original ray of light would go if it didn't refract when it hit the glass.

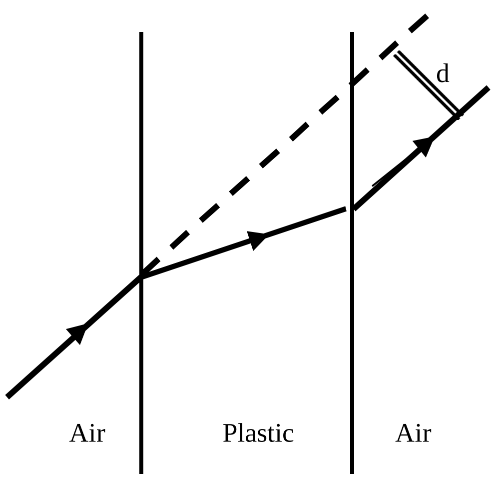


The angle between this "what if" ray and the actual ray that leaves the prism is called the "angle of deviation" and has the symbol δ. If you want to take the time, try to figure out how all the angles relate to find δ. If you don't have that much free time, I'm quite happy if you can just remember the following rule for calculating it:

$$\boxed{\delta = D - (60° - A)}$$

So, in the problem you just did, $\delta = 41°$.

Displacement through Flat Glass

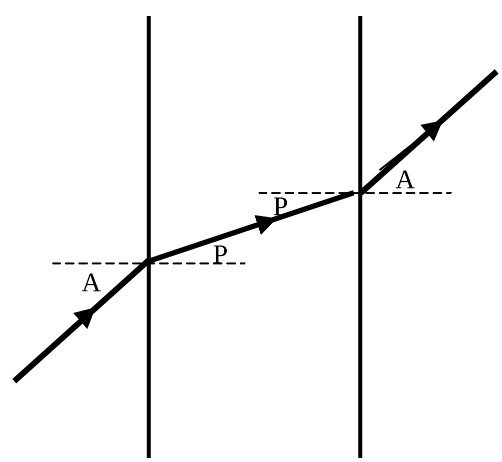


Question #15 on page 365 is a typical one. Light traveling in the air hits a plastic window at a certain angle, refracts inside the plastic, and then exits back into the air on the other side as pictured below.

The dotted line show the path the light would have taken if it didn't refract in the plastic sheet. We want to find how far apart are these two lines, marked as distance "d".

First, figure out what the angle of refraction is inside the plastic (angle P).

Notice that the angles "P" and "A" are each in the diagram twice, since they are equal. As much as the light is refracted as it enters the plastic, it will be refracted just as much as it leaves the plastic.



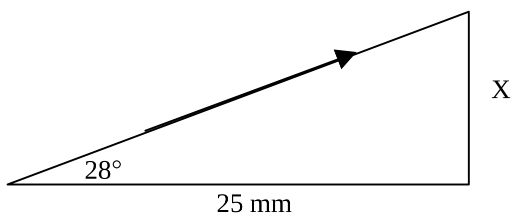
$n_{air} = 1.0$
$n_{plastic} = 1.5$
Angle $A = 45°$
Angle $P = ?$

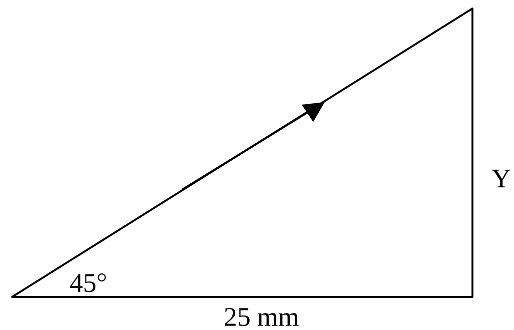
$$(\sin A)\, n_{air} = (\sin P)\, n_{plastic}$$

$$\frac{(\sin 45°)1.0}{1.5} = \sin P$$

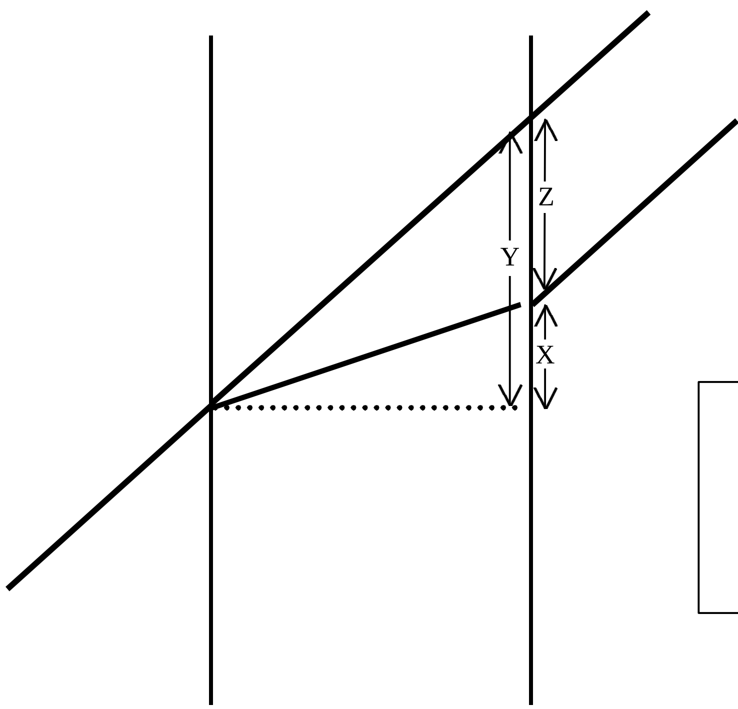
$P = 28°$

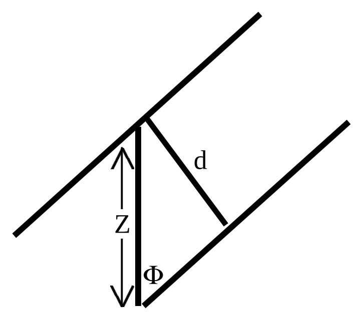
You can now draw a couple of triangles that are inside the plastic. First draw one for the real refracted ray, then one for the ray if it had traveled straight through. Figure out what are the sides “X” and “Y” using tan.





You should get X = 13.3 mm and Y = 25 mm. In the following diagram you can see that Y – X = Z which is along another triangle you can draw, this time outside of the plastic.





Y – X = Z
25 mm – 13.3 mm = 11.7 mm

Since the angle A is 45°, the angle Φ must be
90° - 45° = 45°
Using sin, you can figure out that d = 8.3 mm.